Bell Ringer:

Write the objective statement on your bell ringer sheet located on page 112, then write the function for each translation:

1. P (4,0), P’ (6,0)
   \((x,y) \rightarrow (x+2, y)\)

2. P (0,4), P’ (0,-6)
   \((x,y) \rightarrow (x, y-10)\)

3. P (-4,-4), P’ (4, -4)
   \((x,y) \rightarrow (x+8, y)\)
   \((x,y) \rightarrow (x+2, y-1)\)
Geometry Unit 2
Lesson 9.2 Practice

1. What is the image of point S(-6, 3) under the translation \((x, y) \rightarrow (x-4, y+5)\).
   \((-6, 3) \rightarrow (-6-4, 3+5) = (-10, 8)\)

2. WISP has vertices W(-2, 3), I(5, -1), and S(-4, -2).
   P(-2, -4). WISP is translated 2 units up and 1 unit left.
   a. Plot the pre-image and the image on the graph provided.
   b. Write out the function notation of WISP
      \((x, y) \rightarrow (x-1, y+2)\)
   c. Write out the coordinates of the image of WISP
      \(W'(-3, 5), I'(4, 1), S'(-5, 0), P'(-3, -2)\)

3. Use the figure at the right. (HINT: recall Pythagorean theorem)
   a. Find the length of AB, AC, and BC.
      \[AB = 5\]
      \[BC \approx 6.4 = \sqrt{41}\]
      \[AC = 4\]
   b. Find the length of A'B', A'C', and B'C'.
      \[A'B' = 5\]
      \[B'C' \approx 6.4 = \sqrt{41}\]
      \[A'C' = 4\]
   c. Does the translation preserve congruency?
      yes!
   d. Write the translation rule used.
      \((x, y) \rightarrow (x+1, y-1)\)
Mr. Scott plans another transformation for the Marching Cougars. The sign of the $y$-coordinate of each marcher changes from positive to negative. Maria, whose position is shown by the $X$ in the diagram, moves from point $(2, 4)$ to point $(2, -4)$.

This type of transformation is called a **reflection**. Reflections are sometimes called flips because the figure is flipped like a pancake. On the coordinate plane, examples of reflections are defined by the function $(x, y) \rightarrow (x, -y)$, which is a reflection across the $y$-axis. The example shown above is described by $(x, y) \rightarrow (-x, y)$, which is a reflection across the $y$-axis. The function $(x, y) \rightarrow (y, x)$ defines a reflection across the line $y = x$.

Every reflection has a **line of reflection**, which is the line that the reflection maps to itself. In the above diagram, the line of reflection is the $x$-axis.

1. Complete the table for the two reflections.

<table>
<thead>
<tr>
<th>Pre-image</th>
<th>Image $(x, y) \rightarrow (x, -y)$</th>
<th>Image $(x, y) \rightarrow (-x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1, 10)$</td>
<td>$(1, -10)$</td>
<td>$(-1, 10)$</td>
</tr>
<tr>
<td>$(6, 10)$</td>
<td>$(6, -10)$</td>
<td>$(-6, 10)$</td>
</tr>
<tr>
<td>$(1, 2)$</td>
<td>$(1, -2)$</td>
<td>$(-1, 2)$</td>
</tr>
<tr>
<td>$(6, 2)$</td>
<td>$(6, -2)$</td>
<td>$(-6, 2)$</td>
</tr>
</tbody>
</table>
Lesson 9-3
Reflection

The figure shows an arrow in the coordinate plane. The tip of the arrow is located at point (4, 10).

2. Predict the direction of the arrow after these reflections:
   a. across the x-axis
   b. across the y-axis

3. Draw the two reflections. Were your predictions correct?

4. What reflection maps the arrow to a downward arrow that also has its tip at the point (4, 10)?

5. Could a reflection map the arrow so it points to the right or to the left (i.e., parallel to the x-axis)? If yes, describe the line of reflection.
Mr. Scott arranges the tuba players in an arrow formation, shown below. Then the tuba players undergo a reflection that is described by the function $(x, y) \rightarrow (8 - x, y)$.

6. Draw the reflection. Identify the line of reflection.

7. Which tuba players travel the longest distance during the reflection? Identify this distance.

8. Which tuba player does not travel any distance during the reflection?

9. Explain why the reflection does not change the distance between any given tuba player and the point $(4,5)$.

(Marked: Draw a line after #9)
**Lesson 9-3**

**Reflection**

10. **Use appropriate tools strategically.** Use geometry software to explore reflections. First, use the software to draw a pentagon. Then draw a line of reflection that passes through one side of the pentagon. Use the software to reflect the pentagon over this line of reflection.

11. What happens under this reflection to points that lie on the line of reflection?

12. Use the software to explore how a point not on the line of reflection is related to its image.
   
a. Measure the distance of a point to the line of reflection. Then measure the distance of the point’s image to the line of reflection. What do you find?

b. Draw the segment that connects a point and its image. How is this segment related to the line of reflection?
Lesson 9-3
Reflection

Like other transformations, reflections can be defined independently of the coordinate plane. A reflection is a transformation that maps $P$ to $P'$ across line $\ell$ such that:

- If $P$ is not on $\ell$, then $\ell$ is the perpendicular bisector of $PP'$.
- If $P$ is on $\ell$, then $P = P'$.

To describe reflections, we will use the notation $r_{\ell}(P) = P'$, in which $r_{\ell}$ is the function that maps point $P$ to point $P'$ across line $\ell$, the line of reflection.

13. The diagram shows pentagon $ABCDE$ and the reflection $r_{\ell}$.

- Draw line $\ell$.
- Label the points $r_{\ell}(A) = A'$, $r_{\ell}(B) = B'$, and so on for the five vertices.

14. Quadrilateral $ABDC$ was constructed by drawing scalene triangle $ABC$, then drawing its reflection across line $BC$. Point $P$ is the intersection of $BC$ and $AD$.

- Prove that $BC$ is perpendicular to $AD$.
- Explain why $AB = BD$. 

Chunk: Draw a line after #13

Chunk: Draw a line after #14
9.3 - Reflections KEY.notebook

Lesson 9-3
Reflection

Item 15 is related to the Perpendicular Bisector Theorem. The theorem states that if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment. The converse is also true. If a point is equidistant from two points $A$ and $B$, then it must lie on the perpendicular bisector of $AB$.

Look again at quadrilateral $ABDC$. It was constructed by a reflection of a figure, which means that it has reflectional symmetry. Line segment $BC$ is the line of symmetry of the figure. A line of symmetry can be an actual line in the figure, or it may be imaginary. A figure may have any number of lines of symmetry, including an infinite number.

16. For each figure shown below, draw all of the lines of symmetry.

Triangle $ABC$ has three lines of symmetry, labeled $\ell_1$, $\ell_2$, and $\ell_3$ in the figure below. What can you conclude about the triangle? Explain.

What type of triangle is this? (hint: think about the side lengths)

equilateral
Check Your Understanding

17. Match the figures to their number of lines of symmetry.

- rhombus
- isosceles triangle
- square
- circle

- a. one line
- b. two lines
- c. four lines
- d. infinitely many lines

The figure shows isosceles triangle $ABC$, which by definition has three unequal sides. Point $D$ is on $AC$, and $BD$ is perpendicular to $AC$.

18. Explain why $BD$ is not a line of symmetry for the triangle.

19. Does triangle $ABC$ have any lines of symmetry? Explain.
### Geometry Unit 2

#### 9.3 Vocabulary

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Definition / Naming</th>
<th>Sketch / Symbol / Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reflection</strong></td>
<td>A <strong>reflection</strong> is a transformation in which a figure is flipped over a line, called a line of reflection.</td>
<td>![Reflection Sketch]</td>
</tr>
<tr>
<td><strong>Line of Reflection</strong></td>
<td>In a reflection transformation, a <strong>line of reflection</strong> is the central line about which a figure produces its mirror image.</td>
<td>![Line of Reflection Sketch]</td>
</tr>
<tr>
<td></td>
<td>• It is also called the line of symmetry.</td>
<td></td>
</tr>
<tr>
<td><strong>Reflectional Symmetry</strong></td>
<td>Reflectional symmetry is a figure that has been reflected over a line.</td>
<td>![Reflectional Symmetry Sketch]</td>
</tr>
<tr>
<td><strong>Line of Symmetry</strong></td>
<td>In a reflection transformation, the central line about which a figure produces its mirror image. It is also called the line of reflection.</td>
<td>![Line of Symmetry Sketch]</td>
</tr>
</tbody>
</table>
Lesson 9-3
Reflection

LESSON 9-3 PRACTICE
Use the figure for Items 20 and 21.

20. Draw the reflection of the arrow described by each of these functions, and identify the line of reflection.
   a. \( (x, y) \rightarrow (8 - x, y) \)
   b. \( (x, y) \rightarrow (-2 - x, y) \)
   c. \( (x, y) \rightarrow (x, -y) \)

21. Reason abstractly. Describe a reflection that would map the arrow onto itself.
   Irregular pentagon \( ABCDE \) has exactly one line of symmetry, which passes through point \( A \).

22. What is the image of point \( C \) under a reflection across the line of symmetry?
23. Does any point remain fixed under a reflection across the line of symmetry? Explain.
24. Which sides of the pentagon must be congruent?
HOMEWORK:

page 119 #20-24