



Summer Review Packet  
Plainfield South High School

# AP Calculus BC

**DUE THE FIRST DAY OF SCHOOL**

The problems in this packet are designed to help you review topics from previous mathematics courses that are important to your success in mathematics for the upcoming school year.

Show all work that leads you to each solution on separate sheets of paper. You may use your notes from previous mathematics courses to help you. Additional copies of this packet may be printed from the school's website.

ALL work should be completed and ready to turn in on the FIRST DAY of school.

**ENJOY YOUR SUMMER!! WE ARE LOOKING FORWARD TO  
SEEING YOU IN THE FALL.**

Student Name \_\_\_\_\_

Student Signature \_\_\_\_\_

Math Course Taken Last Year: \_\_\_\_\_

Last Year's Math Teacher: \_\_\_\_\_

## AP Calculus BC Summer Assignment

Attached is an assignment for students entering AP Calculus BC in the fall. Next year we will focus more on concepts and thinking outside of the box. We will not have time to review basic algebraic concepts that you should already know. Therefore it is imperative that you have mastered the basic skills needed for success in this course. Many students struggle with the Algebra component of Calculus and neglect the concepts and theorems. It is a shame when a student understands the calculus but can't simplify, thus getting the problem incorrect. Thus the purpose of this worksheet is to help you be better prepared for next year.

Also included are basic concepts you must know. You may want to use your notes from Pre-Calculus as well. Although the assignment says BC, the introductory concepts are the same.

**YOU MUST COMPLETE THE FOLLOWING PROBLEMS ON A SEPARATE SHEET OF PAPER**

**#1-77, 79, 80, 83-132, 134.**

This will be checked on the first day of school. I will be checking for completeness and correctness.

**There should be evidence that you have checked your work.** A key will be provided (e-mailed). Put an **X** by the problems that you got wrong but corrected. Put a **?** by the problems you got wrong but had difficulty correcting. There will be a **TEST** on this material the first week of school!

You should wait till mid-summer to begin this packet. This will give you a better indication of what you know and what you've forgotten. If you have any questions, contact me at [tsingler@psd202.org](mailto:tsingler@psd202.org). Keep in mind that this packet is not to "torture" you or "punish" you for taking AP Calculus BC. This will better prepare you and allow us more time to review before the exam.

**Enjoy your summer,**

**Mrs. Singler**

AP CALCULUS BC SUMMER ASSIGNMENT

Name \_\_\_\_\_

Complete the following. Show and attach all work in a clear manner. Do NOT do any work on this sheet – all work must be separate. Have this assignment completed and ready to turn in on the first day of school. No calculator except were noted.

1 – 6. Are the following statements true? If not, explain in words why not?

1.  $\frac{2k}{2x+h} = \frac{k}{x+h}$

2.  $\frac{1}{p+q} = \frac{1}{p} + \frac{1}{q}$

3.  $\frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

4.  $3\frac{a}{b} = \frac{3a}{3b}$

5.  $3\frac{a}{b} = \frac{3a}{b}$

6.  $3\frac{a+b}{c} = \frac{3a+b}{c}$

7 – 16. Factor each of the following completely.

7.  $a^2 - b^2$

8.  $a^3 - b^3$

9.  $8x^3 + y^3$

10.  $4x^2 - 21x - 18$

11.  $2x^2 + x - 3$

12.  $3x^2 + 6x^3 - 9x$

13.  $(x+1)^3(4x-9) - (16x+9)(x+1)^2$

14.  $(x-1)^3(2x-3) - (2x+12)(x-1)^2$

15.  $(2x-1)^2(x-3) + (x+1)(2x-1)^3$

16. Factor  $x - a$  in such a way that  $\sqrt{x} - \sqrt{a}$  is a factor.

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16. Factor  $x - a$  in such a way that  $\sqrt{x} - \sqrt{a}$  is a factor.

17 - 24. Simplify:

17.  $\frac{\frac{x}{2}}{\frac{x}{4}}$

18.  $h \div \frac{(x+h)}{h}$

19.  $\frac{\sqrt{x-2} + \frac{5}{\sqrt{x-2}}}{x-2}$

20.  $\frac{x(5x+1) - 3(x^2+1)}{(x-1)^2}$

21.  $\frac{(x+1)^3(4x-9) - (16x+9)(x+1)^2}{(x-6)(x+1)^3}$

22.  $\frac{3x(x+1) - 2(2x+1)}{(x-1)^2}$

23.  $\frac{2x(x+1)^2 - 3(x+1)^3}{8x^3 + 30x^2 + 18x}$

24.  $\frac{(x-1)^3(2x-3) - (4x-1)(x-1)^2}{(x-1)^2(2x-1)}$

25 – 28. Solve the equation.

25.  $4x^2 - 21x - 18 = 0$

26.  $x^3 + 3x^2 - 5x - 15 = 0$

27.  $x^4 - 9x^2 + 8 = 0$

28.  $4 - 3^x = 0$

29. Write as a single fraction with denominator in factored form:  $\frac{7x^2 + 5x}{x^2 + 1} - \frac{5x}{x^2 - 6} = 0$

30. Graph the equation  $y = x^3 - x$  and answer the following questions.

- Is the point (3, 2) on the graph?
- Is the point (2, 6) on the graph?
- Is the function even, odd, or neither?
- What is the y intercept?
- Find the x intercepts.

31. Find all intercepts of the graph of  $y = \frac{x-1}{x+3}$

32 – 35. Show work to determine if the relation is even, odd, or neither.

32.  $f(x) = 2x^2 - 7$

33.  $f(x) = -4x^3 - 2x$

34.  $f(x) = 4x^2 - 4x + 4$

35.  $f(x) = \frac{x^2}{x^2 - 4}$

36. Find the equation of the straight line that passes through the point (2, 4) and is parallel to the line  $2x + 3y - 8 = 0$ .

37. Find the equation of the line that is perpendicular to the line  $2x + 3y - 8 = 0$  at the point (1,2)

38. The line with the slope 5 that passes through the point (-1,3) intersects the x axis at a point. What are the coordinates of this point?

39. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2,4) intersects the y axis?

40. Given  $f(x) = |x - 3| - 5$  find  $f(1) - f(5)$ .

41. Find all points of intersection of the graphs of  $x^2 + 3x - y = 3$  and  $x + y = 2$

42. If the point  $(-1, 1)$  lies on the graph of the equation  $kx^2 - xy + y^2 = 5$ , find the value of k.

43. Write the equation of a graph that is a function.

44. Write the equation of a graph that is not a function.

44. Write the equation of a graph that is not a function.

45 – 48. Find the domain for each of the following functions.

45.  $h(x) = \frac{1}{4x^2 - 21x - 18}$

46.  $k(x) = \sqrt{x^2 - 5x - 14}$

47.  $p(x) = \frac{\sqrt[3]{x-6}}{\sqrt{x^2 - x - 30}}$

48.  $y = \ln(2x - 12)$

49. For the function  $y = 5 - \sqrt{9 - x^2}$ , a) find the domain, b) find the range, and c) determine whether the function is odd, even, or neither.

50. Let  $f(x) = \begin{cases} -0.5x & x < -2 \\ \sqrt{x+2} & x \geq -2 \end{cases}$  a) draw the graph of  $f(x)$ . b) find the domain, c) find the range.

51. Find  $f(x + \Delta x)$  for  $f(x) = x^2 - 2x - 3$ .

52 – 53. Sketch the graph of each function

52.  $f(x) = \begin{cases} 1 & x \leq 0 \\ -1 & x > 0 \end{cases}$

53.  $f(x) = \begin{cases} 2x & (-\infty, -1) \\ 2x^2 & [-1, 2) \\ -x + 3 & (2, \infty) \end{cases}$

54. State the domain, range and intercepts of the function  $y = 2^{-x} - 1$ .

55 – 57. Given  $f(x) = x - 3$  and  $g(x) = \sqrt{x}$  complete the following

55.  $f(g(x)) =$                       56.  $g(f(x)) =$                       57.  $f(f(x)) =$

58 – 60. Given  $f(x) = \frac{1}{x-5}$  and  $g(x) = x^2 - 5$  complete the following

58.  $f(g(7)) =$                       59.  $g(f(v)) =$                       60.  $g(g(x)) =$

61 – 64. Let  $f(x) = 2x - 2$ . Complete the following:

61. Sketch the graph of  $f(x)$ .                      62. Determine whether  $f$  has an inverse function.

63. Sketch the graph of  $f^{-1}(x)$                       64. Give the equation for  $f^{-1}(x)$

65 – 66. Simplify using only positive exponents. Do not rationalize the denominator.

65.  $\frac{\sqrt{4x-16}}{\sqrt[3]{(x-4)^3}}$                       66.  $\left(\frac{1}{x^2} + \frac{4}{x^{-1}y^{-1}} + \frac{1}{y^{-2}}\right)^{-\frac{1}{2}}$

67 – 72. If  $f(x) = x^2 - 1$ , describe in words what the following would do to the graph of  $f(x)$ .

67.  $f(x) - 4$                       68.  $f(x - 4)$                       69.  $-f(x + 2)$

70.  $5f(x) + 3$                       71.  $f(2x)$                       72.  $|f(x)|$

73. (calculator) The dollar value of a product in 1998 is \$78. The value of the product is expected to decrease \$5.75 per year for the next 5 years. Write a linear equation that gives the dollar value  $V$  of the product in terms of the year  $t$ . (Let  $t = 8$  represent 1998).

74. (calculator) A business had annual retail sales of \$124,000 in 1993 and \$211,000 in 1996. Assuming that the annual increase in sales follows a linear pattern, predict the retail for 2007.

75. (calculator) In order for a company to realize a profit in the manufacture and sale of a certain item, the revenue,  $R$ , for selling  $x$  items must be greater than the cost,  $C$ , of producing  $x$  items. If  $R = 69.99x$  and  $C = 59x + 850$ , for what values of  $x$  will this product return a profit?

76. (calculator) Suppose that in any given year, the population of a certain endangered species is reduced by 25%. If the population is now 7500, in how many years will the population be 4000?

77. A piece of wire 5 inches long is to be cut into two pieces. One piece is  $x$  inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of  $x$ .

78. a) Graph the parametrized curve described by  $x = 2\sin t$ ,  $y = -3\cos t$ ,  $0 \leq t \leq \pi$ . Indicate the direction in which the curve is traced. b) Find a Cartesian equation for the parametrized curve. What portion of the graph of the Cartesian equation is traced by the parametrized curve?

79. Let  $f(x) = \sqrt[3]{x+2}$  and  $g(x) = x^3 - 2$ . Which of the following are true?

- I.  $g(x) = f^{-1}(x)$  for all real values of  $x$ .
- II.  $(f \circ g)(x) = 1$  for all real values of  $x$ .
- III. The function  $f$  is one to one

80. Let  $f(x) = \sqrt{3-x}$ . Find an expression for  $f^{-1}(x)$ . (Be sure to state any necessary domain restrictions.)

81. (calculator) The table gives Taiwan's nuclear power generation data in billions of kilowatt-hours. Let  $x = 5$  represent 1980,  $x = 10$  represent 1985, and so on.

Year	1980	1985	1990	1995
Energy produced	7.8	27.8	31.6	33.9

- a. Find a natural logarithm regression equation for the data.
- b. predict when Taiwan's nuclear power generation will reach 40 billion kilowatt-hours.

82. An angle measuring  $\frac{3\pi}{8}$  radians has its vertex at the center of a circle whose radius is 4 feet. Find the length of the subtended arc.

83. Let  $y = 3\sin(2x - \pi) + 2$ . Determine the period, domain, and range of the function.

84 – 91. Evaluate: Answers for 90 & 91 must be in radians.

- |                |                                  |                          |                          |                          |
|----------------|----------------------------------|--------------------------|--------------------------|--------------------------|
| 84. $\cos 0$   | 85. $\sin 0$                     | 86. $\tan \frac{\pi}{2}$ | 87. $\cos \frac{\pi}{4}$ | 88. $\sin \frac{\pi}{2}$ |
| 89. $\sin \pi$ | 90. $\arccos \frac{\sqrt{3}}{2}$ | 91. $\arctan 1$          |                          |                          |

92 – 94. Find the solution of the equations for  $0 \leq x < 2\pi$

- |  |  |                                      |
|--|--|--------------------------------------|
| 92. $2\sin^2 \theta = 1 - \sin \theta$ | 93. $2\tan \theta - \sec^2 \theta = 0$ | 94. $\sin 2\theta + \sin \theta = 0$ |
|--|--|--------------------------------------|

95. Which of the following expressions are identical?

- a)  $\cos^2 x$       b)  $(\cos x)^2$       c)  $\cos x^2$

96. Which of the following expressions are identical?

- a)  $(\sin x)^{-1}$       b)  $\arcsin x$       c)  $\sin x^{-1}$       d)  $\frac{1}{\sin x}$

97 – 103. Solve for  $x$ .

- |                                       |                          |                                  |
|---------------------------------------|--------------------------|----------------------------------|
| 97. $\ln e^3 = x$                     | 98. $\ln e^x = 4$        | 99. $\ln x + \ln x = 0$          |
| 100. $e^{\ln 6} = x$                  | 101. $\ln 1 - \ln e = x$ | 102. $\ln 6 + \ln x - \ln 2 = 3$ |
| 103. $\ln(x+5) = \ln(x-1) - \ln(x+1)$ |                          |                                  |

104 – 121: Evaluate the limit.

104.  $\lim_{x \rightarrow 2} (3x^2 + 5)$

105.  $\lim_{x \rightarrow -1} \frac{x^4 + x^3}{x + 1}$

106.  $\lim_{x \rightarrow 25} \frac{\sqrt{x} - 5}{x - 25}$

107.  $\lim_{x \rightarrow -2} \frac{x - 4}{x^2 - 2x - 8}$

108.  $\lim_{x \rightarrow -2} \frac{x^3 + 8}{x + 2}$

109.  $\lim_{x \rightarrow 5} \frac{x - 5}{|x - 5|}$

110.  $\lim_{x \rightarrow 1} f(x)$  if  $f(x) = \begin{cases} 3 - x & x \neq 1 \\ 1 & x = 1 \end{cases}$

111.  $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x^2 + 1}$

112.  $\lim_{x \rightarrow 2} \sqrt{x^2 - 4}$

113. If  $\lim_{x \rightarrow c} f(x) = -\frac{1}{2}$ , and  $\lim_{x \rightarrow c} g(x) = \frac{2}{3}$ , find  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)}$

114.  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

115.  $\lim_{x \rightarrow 2} \sec \frac{\pi x}{3}$

116.  $\lim_{x \rightarrow 0} \frac{x}{\tan x}$

117.  $\lim_{x \rightarrow 3^-} \sqrt{2x - 5}$

118.  $\lim_{x \rightarrow 2^-} \frac{1}{x - 2}$

119.  $\lim_{x \rightarrow 2} \frac{1}{(x - 2)^2}$

120.  $\lim_{x \rightarrow 0} \left( 2 + \frac{5}{x^2} \right)$

121.  $\lim_{x \rightarrow \infty} \frac{x^2 - x - 6}{5x^2}$

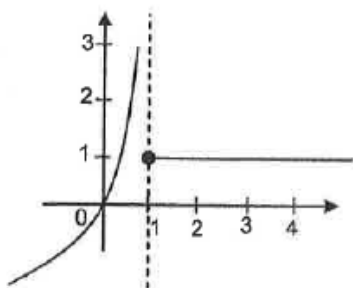
122 – 127. For each of the following determine: a)  $\lim_{x \rightarrow c^-} f(x)$  b)  $\lim_{x \rightarrow c^+} f(x)$  and c)  $\lim_{x \rightarrow c} f(x)$

122.  $f(x) = \begin{cases} x^2 - 1 & x < 1 \\ 4 - x & x \geq 1 \end{cases}$

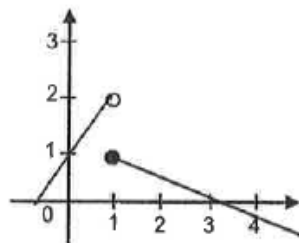
123.  $f(x) = \begin{cases} 3x - 1 & x \leq 1 \\ 3 - x & x > 1 \end{cases}$

124.  $f(x) = \begin{cases} -x^2 & x < 1 \\ 2 & x = 1 \\ x - 2 & x > 1 \end{cases}$

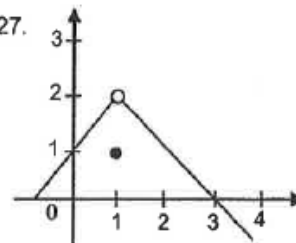
125.



126..



127.



128 – 13. Answer the following.

128. At which values of  $x$  is  $f(x) = \frac{(x+1)(x-3)}{x-2}$  discontinuous?

129. Let  $f(x) = \frac{1}{x+1}$  and  $g(x) = x^2 - 5$ . Find all values of  $x$  for which  $f(g(x))$  is discontinuous.

130. Determine the value of "c" so that  $f(x)$  is continuous on the entire real line.  $f(x) = \begin{cases} x-2 & x \leq 5 \\ cx-3 & x > 5 \end{cases}$

131. Find all vertical asymptotes of  $f(x) = \frac{x-3}{x+2}$ .

132. Find all vertical asymptotes of  $g(x) = \frac{x+1}{x^2-1}$ .

133.  $f(x)$  decreases without bound as  $x$  approaches what value from the right?  $f(x) = \frac{4}{(x-3)(5-x)}$

134. Find the derivative of  $y = 5x^3 - 7x^2 + 2x - 9$ .