

Discrete Mathematics
Semester 1 Final Exam Review

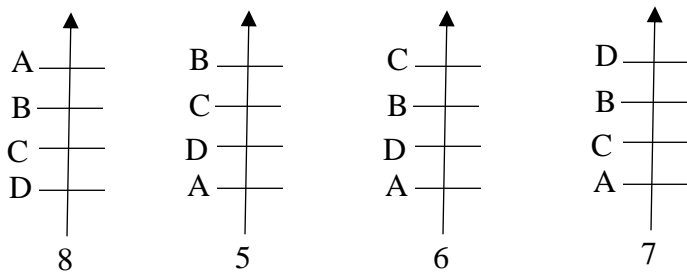
Name _____

Semester 1 Outcomes:

- A: Election Theory
- B: Logic and Set Theory
- C: Fair Division
- D: Matrices
- E and F: Graph Theory

Outcome A: Election Theory

Use the following preference schedule to answer questions 1 – 5



1. Determine the plurality winner? Is it also a majority winner? Explain.

2. Determine the Borda winner.

3. Determine the runoff winner.

4. Determine the sequential runoff winner.

5. Suppose that this election is conducted by an approval method and all voters approve of their first two choices on their preference schedules. Determine the winner using the approval method.

6. Determine a Condorcet winner.

Consider a situation in which voters A gets 23% of the vote, B gets 30% of the vote, C gets 28% of the votes, and D gets 19% of the votes. 50% of the votes are needed to pass an issue. Use the following coalitions to answer questions 7-9.

I. {A, D: 42}

II. {A, B: 53}

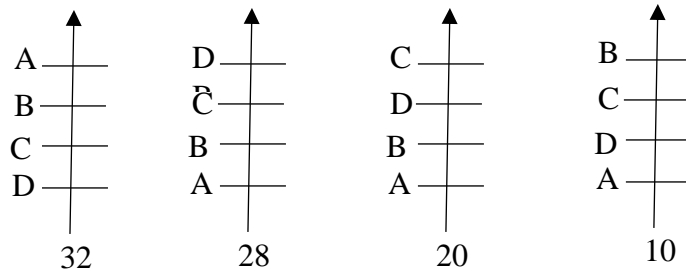
III. {B, C, D: 77}

7. List all winning coalitions?

8. Find the power index for each voter.

9. Which voter has the most power in the votes? Which voter has the least power in the votes?

10. Choose an election method that you think is best to use to determine a winner for the following preferences. Explain why you think your method of choice is best.



11. Consider a situation in which voters A, B, C, and D have 4, 3, 3, and 2 votes, respectively, and 7 votes are needed to pass an issue. List all winning coalitions and their vote totals.

Outcome B: Logic and Set Theory

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{1, 3, 5, 7, 9\}$. Describe each of the following sets for questions 1-5.

1. $A \cup B$

2. $B \cup C$

3. $A \cap C$

4. $\bar{A} \cap \bar{B}$

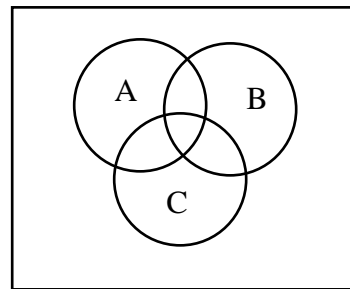
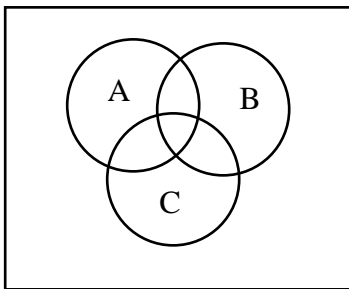
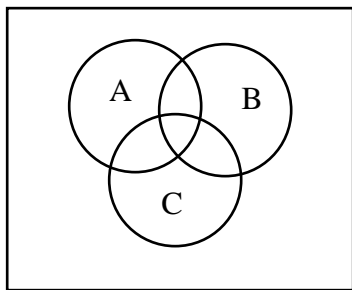
5. $(A \cup C) \cap B$

For questions 6-8, shade the portion of the Venn diagram that illustrates each of the following sets.

6. $\bar{B} \cap C$

7. $A \cup B$

8. $A \cap \bar{B} \cap C$



9. Let $A = \{x, y, z\}$ and $B = \{1, 2, 3\}$. List the elements of each of the following Cartesian products.

a. $A \times B$

b. $B \times A$

10. Let $A = \{4, 5, 7, 9, 11\}$

$B = \{9, 11, 13\}$

$C = \{4, 5, 9\}$

$D = \{11, 13\}$

State whether the following are TRUE or FALSE

(a) $C \subset A$ _____

(b) $B \subset A$ _____

(c) $D \subset A$ _____

(d) $A \not\subset B$ _____

(e) $B \not\subset C$ _____

(f) $D \not\subset B$ _____

2. Lynne, Pauline, and Tim have just learned that they are heirs to the estate of their recently deceased Uncle George. The only items of value in the estate are a guitar, a car, a kayak, and an expensive watch. Lynn, Pauline, and Tim submit bids as shown in the following table.

	Lynn	Pauline	Tim
Guitar	\$1,500	\$2,500	\$2,200
Car	\$6,000	\$5,500	\$5,500
Kayak	\$700	\$200	\$600
Watch	\$250	\$400	\$350

Lynn was always George's favorite, and so his will states that Lynn should receive half of his estate and that Pauline and Tim should each receive a quarter. For each heir, find their fair share, the items received, the amount of cash, and the final settlement.

3. States A, B, and C have populations of 648, 247, and 105, respectively. There are 100 seats to apportion among them.

- a.) What is the ideal ratio?
- b.) Find the quota for each state.
- c.) Apportion the 100 seats among the three states by the Hamilton method.
- d.) What is the initial Jefferson apportionment?
- e.) Find the Jefferson adjusted ratio for each state.
- f.) Apportion the 100 states by the Jefferson method.

4. A cookie cake is divided among 10 players ($P_1, P_2, P_3, \dots, P_{10}$) using the Last-Diminisher Method. The players play in a fixed order, with P_1 first, P_2 second and so on.

Round 1: After the first cut is made... P_3, P_5 and P_7 are the only diminishers.

Round 2: After the first cut is made... P_4 and P_8 are the only diminishers.

Round 3: After the first cut is made... all players pass

Who gets the piece of the cookie cake after each of the first 3 rounds?

Outcome D: Matrices

1. A trendy garment company receives orders from three clothing shops. The first shop orders 25 jackets, 75 shirts, and 75 pairs of pants. The second shop orders 30 jackets, 50 shirts, and 50 pairs of pants. The third shop orders 20 jackets, 40 shirts, and 35 pairs of pants. Display this information in a matrix. Let the rows represent the shops and the columns represent the type of garment ordered. Label the rows and columns of your matrix accordingly.

2. Matrix A shows the prices of three different pizza places. $A = \begin{matrix} & \text{Vin's} & \text{Toni's} & \text{Sal's} \\ \text{Pizza} & \$10.10 & \$10.86 & \$10.65 \\ \text{Drinks} & \$1.09 & \$0.89 & \$1.05 \\ \text{Salad} & \$3.69 & \$3.89 & \$3.85 \end{matrix}$

a.) What is the value of A_{21} ?

b.) What is the value of A_{12} ?

For questions 3-6, use the following matrices to evaluate.

$$A = \begin{bmatrix} -7 & 16 \\ x & 8 \end{bmatrix} \quad B = \begin{bmatrix} 0 & y \\ 10 & -6 \\ 11 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 7 \\ -10 & w \end{bmatrix} \quad D = \begin{bmatrix} -2 & 0 & 13 \\ z & -9 & 6 \\ -6 & 2 & 3 \end{bmatrix}$$

3. $2A + 4C$

4. $3C - A$

5. BC

6. $2D$

7. Dad's Donuts sell the following types of donuts on Saturday morning: 191 plain for \$0.45, 122 jelly for \$0.55, 98 glazed for \$0.50, and 69 specialty for \$0.85. Write a row matrix to represent the cost of the donuts and a column matrix to represent how many they sold. Then find the total sales for the day using matrix operations.

8. Birth rates and age specific survival rates for 3-month periods are summarized in the following table.

Age (months)	Birth Rate	Survival Rate
0-3	0	0.6
3-6	0.3	0.9
6-9	0.8	0.9
9-12	0.7	0.8
12-15	0.4	0.6
15-18	0	0

Suppose the original female rat population is 42 animals with the age distribution shown in the following table.

Age (months)	0-3	3-6	6-9	9-12	12-15	15-18
Number	15	9	13	5	0	0

Find the population total and distribution after 3 months (1 cycle) for the following initial populations.

a.) $[35 \ 0 \ 0 \ 0 \ 0 \ 0]$

b.) $[5 \ 5 \ 5 \ 5 \ 5 \ 5]$

9. The students at Central High are planning to hire a band for the prom. Their choices are bands A, B, and C. They survey the Sophomore, Junior, and Senior classes and find the following percentages of students (regardless of sex) prefer the bands,

$$\begin{matrix} & 10^{\text{th}} & 11^{\text{th}} & 12^{\text{th}} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 20\% & 35\% & 40\% \\ 30\% & 30\% & 25\% \\ 50\% & 35\% & 35\% \end{bmatrix} \end{matrix}$$

The student population by class and sex is:

$$\begin{matrix} & \text{Male} & \text{Female} \\ \begin{matrix} 10^{\text{th}} \\ 11^{\text{th}} \\ 12^{\text{th}} \end{matrix} & \begin{bmatrix} 235 & 225 \\ 205 & 215 \\ 175 & 190 \end{bmatrix} \end{matrix}$$

Use matrix multiplication to find:

- The number of males and females who prefer each band.
- The total number of students who prefer each band.

10. The characteristics of the female population of a herd of small mammals are shown in the following table.

Age groups (months)

	0-4	4-8	8-12	12-16	16-20	20-24
Birth Rate	0	0.5	1.1	0.9	0.4	0
Survival Rate	0.6	0.8	0.9	0.8	0.6	0

Suppose the initial female population for the herd is given by $P_0 = [22 \ 22 \ 18 \ 20 \ 7 \ 2]$

- a.) What is the expected lifespan of this mammal?
- b.) Construct the Leslie matrix for this population.
- c.) Determine the long-term growth rate for the herd.
- a.) How many mammals will there be after 4 years?

Outcome E and F: Graph Theory

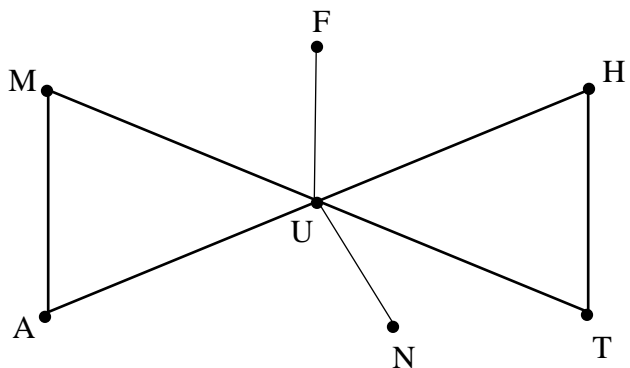
1. Describe the route that is used for each of the following:

- a.) Euler Circuit
- b.) Euler Path
- c.) Hamiltonian Path
- d.) Hamiltonian Path

2. Use the picture to the right.

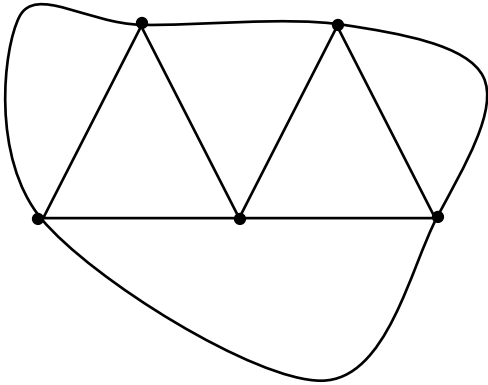
What is the valence of each of the following:

- a.) H
- b.) F

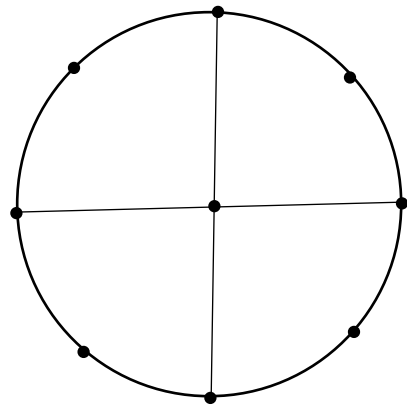


3. State whether each graph has an Euler circuit, Euler path, or neither. Explain why.

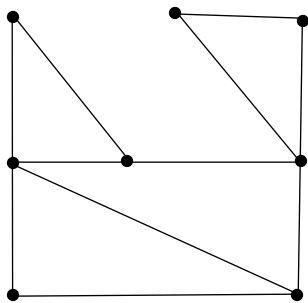
a.)



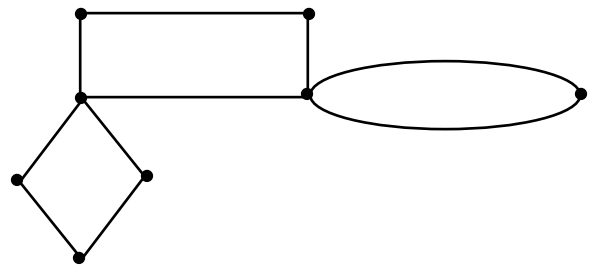
b.)



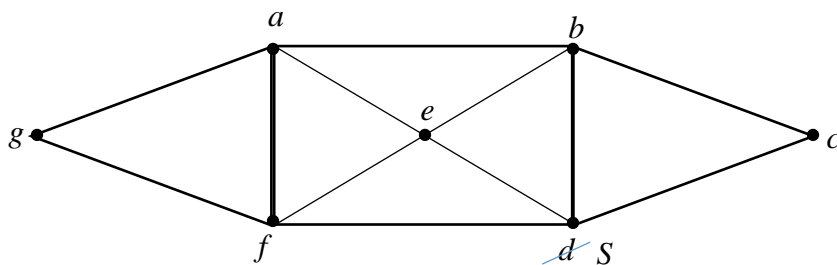
c.)



d.)



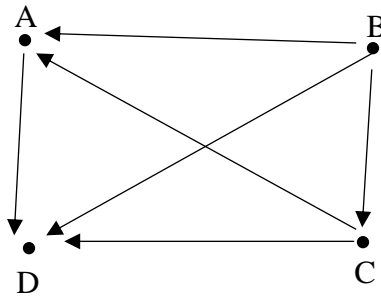
4. Sally began using the Euler circuit algorithm to find the Euler circuit for the following graph. She started at vertex d and labeled it S . The first circuit she found was S, e, f, a, b, c, S . Using Sally's start, continue the algorithm and find an Euler circuit for the graph.



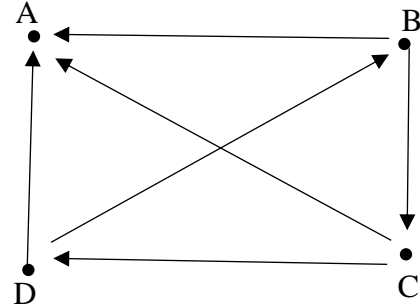
5. Draw a graph with 6 vertices and 12 edges as an Euler Circuit.

6. Find all the directed Hamiltonian paths for each of the following tournaments:

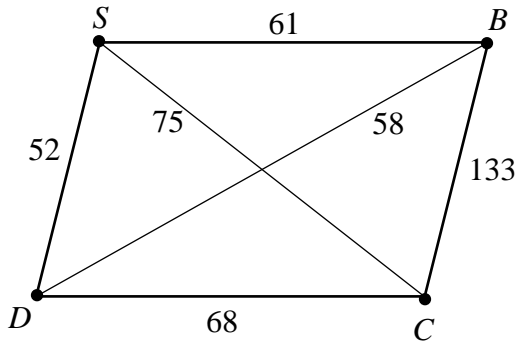
a.)



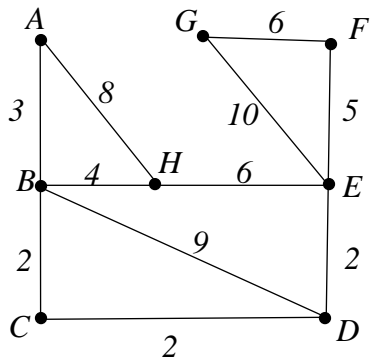
b.)



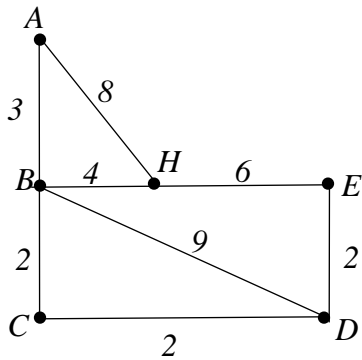
7. Using the nearest-neighbor algorithm, what is the length of the route which begins and ends at S, visiting each other city exactly once? Does it produce the optimal solution?



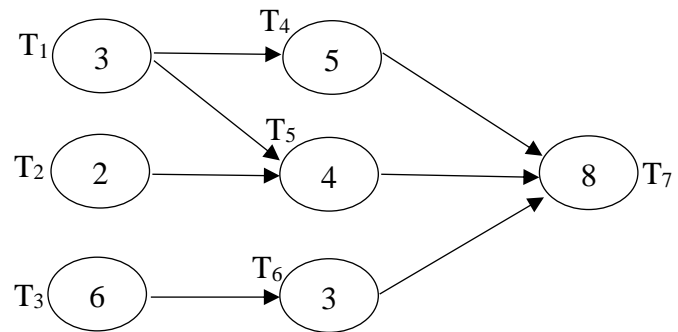
8. Use the shortest path algorithm to find the shortest route from A to F.



9. Use the Traveling Salesman method to determine the minimum distance for a business trip leaving from H. What is the minimum distance?



10. Below is a graph representing the tasks necessary to build a new car with the accompanying times in hours. What is the length of the critical path?



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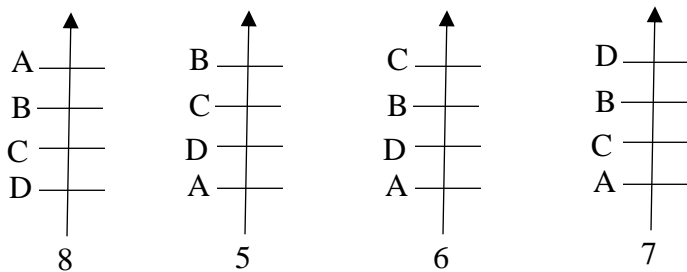
Name _____ **KEY** _____

Semester 1 Outcomes:

- A: Election Theory
- B: Logic and Set Theory
- C: Fair Division
- D: Matrices
- E and F: Graph Theory

Outcome A: Election Theory

Use the following preference schedule to answer questions 1 – 5



1. Determine the plurality winner? Is it also a majority winner? Explain.

Plurality winner is A. It is not a majority winner because they had 8 first place votes and need more than 13 votes to be a majority winner.

2. Determine the Borda winner.

Borda winner is B

3. Determine the runoff winner.

Runoff winner is D

4. Determine the sequential runoff winner.

Sequential runoff winner is C

5. Suppose that this election is conducted by an approval method and all voters approve of their first two choices on their preference schedules. Determine the winner using the approval method.

Approval winner is B

6. Determine a Condorcet winner.

Condorcet winner is B

Consider a situation in which voters A gets 23% of the vote, B gets 30% of the vote, C gets 28% of the votes, and D gets 19% of the votes. 50% of the votes are needed to pass an issue. Use the following coalitions to answer questions 7-9.

I. {A, D: 42}

II. {A, B: 53}

III. {B, C, D: 77}

7. List all winning coalitions?

AB, AC, BC, ABC, ABD, ACD, BCD, ABCD

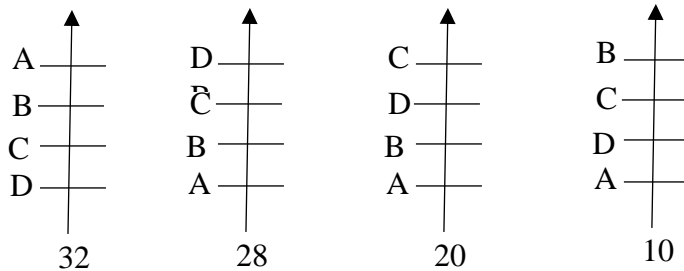
8. Find the power index for each voter.

A-4 B-4 C-4 D-0

9. Which voter has the most power in the votes? Which voter has the least power in the votes?

No single voter has the most power but voter D has NO power, and is considered a dummy voter.

10. Choose an election method that you think is best to use to determine a winner for the following preferences. Explain why you think your method of choice is best.



ANSWERS MAY VARY

11. Consider a situation in which voters A, B, C, and D have 4, 3, 3, and 2 votes, respectively, and 7 votes are needed to pass an issue. List all winning coalitions and their vote totals.

AB 7, AC 7, ABC 10, ABD 9, ACD 9, BCD 8, ABCD 12

Outcome B: Logic and Set Theory

Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$; $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{1, 3, 5, 7, 9\}$. Describe each of the following sets for questions 1-5.

1. $A \cup B$

$\{1, 2, 3, 4, 5, 6, 8, 10\}$

2. $B \cup C$

$\{1, 2, 3, 4, 5, 7, 9\}$

3. $A \cap C$

$\{\emptyset\}$

4. $\bar{A} \cap \bar{B}$

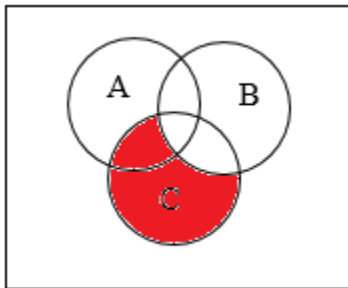
$\{7, 9\}$

5. $(A \cup C) \cap B$

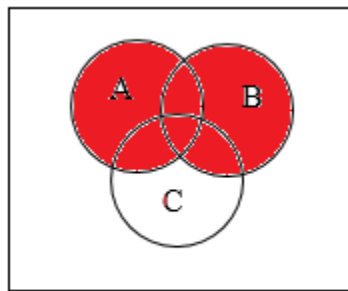
$\{1, 2, 3, 4, 5\}$

For questions 6-8, shade the portion of the Venn diagram that illustrates each of the following sets.

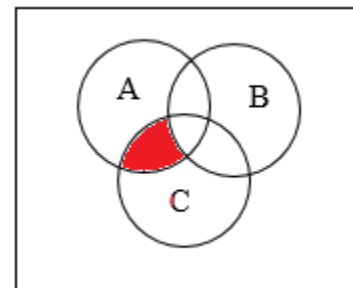
6. $\bar{B} \cap C$



7. $A \cup B$



8. $A \cap \bar{B} \cap C$



9. Let $A = \{x, y, z\}$ and $B = \{1, 2, 3\}$. List the elements of each of the following Cartesian products.

a. $A \times B$

$\{(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3), (z, 1), (z, 2), (z, 3)\}$

b. $B \times A$

$\{(1, x), (1, y), (1, z), (2, x), (2, y), (2, z), (3, x), (3, y), (3, z)\}$

10. Let $A = \{4, 5, 7, 9, 11\}$

$B = \{9, 11, 13\}$

$C = \{4, 5, 9\}$

$D = \{11, 13\}$

State whether the following are TRUE or FALSE

(a) $C \subset A$ True

(b) $B \subset A$ False

(c) $D \subset A$ False

(d) $A \not\subset B$ True

(e) $B \not\subset C$ True

(f) $D \not\subset B$ True

2. Lynne, Pauline, and Tim have just learned that they are heirs to the estate of their recently deceased Uncle George. The only items of value in the estate are a guitar, a car, a kayak, and an expensive watch. Lynn, Pauline, and Tim submit bids as shown in the following table.

	Lynn	Pauline	Tim
Guitar	\$1,500	\$2,500	\$2,200
Car	\$6,000	\$5,500	\$5,500
Kayak	\$700	\$200	\$600
Watch	\$250	\$400	\$350

Lynn was always George's favorite, and so his will states that Lynn should receive half of his estate and that Pauline and Tim should each receive a quarter. For each heir, find their fair share, the items received, the amount of cash, and the final settlement.

Lynn: fair share is \$4,225; gets car and kayak and pays Tim \$2,120.83

Pauline: fair share is \$2,150; gets guitar and watch and pays Tim \$395.83

Tim: fair share \$2,162.50; gets \$2, 516.67

3. States A, B, and C have populations of 648, 247, and 105, respectively. There are 100 seats to apportion among them.

- What is the ideal ratio? **10**
- Find the quota for each state. **A: 64.8, B: 24.7, C: 10.5**
- Apportion the 100 seats among the three states by the Hamilton method. **A:66, B: 24, C: 10**
- What is the initial Jefferson apportionment? **A: 64, B: 24, C: 10**
- Find the Jefferson adjusted ratio for each state. **A: 9.97, B: 9.88, C: 9.55**
- Apportion the 100 states by the Jefferson method. **A:66, B: 24, C: 10**

4. A cookie cake is divided among 10 players ($P_1, P_2, P_3, \dots, P_{10}$) using the Last-Diminisher Method. The players play in a fixed order, with P_1 first, P_2 second and so on.

Round 1: After the first cut is made... P_3, P_5 and P_7 are the only diminishers.

Round 2: After the first cut is made... P_4 and P_8 are the only diminishers.

Round 3: After the first cut is made... all players pass

Who gets the piece of the cookie cake after each of the first 3 rounds? **Round 1: P_7 gets the cake, Round 2: P_8 gets the cake, Round 3: P_1 gets the cake**

Outcome D: Matrices

1. A trendy garment company receives orders from three clothing shops. The first shop orders 25 jackets, 75 shirts, and 75 pairs of pants. The second shop orders 30 jackets, 50 shirts, and 50 pairs of pants. The third shop orders 20 jackets, 40 shirts, and 35 pairs of pants. Display this information in a matrix. Let the rows represent the shops and the columns represent the type of garment ordered. Label the rows and columns of your matrix accordingly.

$$A = \begin{matrix} & \begin{matrix} \text{Jackets} & \text{Shirts} & \text{Pants} \end{matrix} \\ \begin{matrix} \text{Shop 1} \\ \text{Shop 2} \\ \text{Shop 3} \end{matrix} & \begin{bmatrix} 25 & 75 & 75 \\ 30 & 50 & 50 \\ 20 & 40 & 35 \end{bmatrix} \end{matrix}$$

2. Matrix A shows the prices of three different pizza places. $A = \begin{matrix} & \text{Vin's} & \text{Toni's} & \text{Sal's} \\ \text{Pizza} & \$10.10 & \$10.86 & \$10.65 \\ \text{Drinks} & \$1.09 & \$0.89 & \$1.05 \\ \text{Salad} & \$3.69 & \$3.89 & \$3.85 \end{matrix}$

a.) What is the value of A_{21} ? **\$1.09**

b.) What is the value of A_{12} ? **\$10.86**

For questions 3-6, use the following matrices to evaluate.

$$A = \begin{bmatrix} -7 & 16 \\ x & 8 \end{bmatrix} \quad B = \begin{bmatrix} 0 & y \\ 10 & -6 \\ 11 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 8 & 7 \\ -10 & w \end{bmatrix} \quad D = \begin{bmatrix} -2 & 0 & 13 \\ z & -9 & 6 \\ -6 & 2 & 3 \end{bmatrix}$$

3. $2A + 4C$

4. $3C - A$

$$\begin{bmatrix} 18 & 60 \\ 2x - 40 & 4w + 16 \end{bmatrix}$$

$$\begin{bmatrix} 31 & 5 \\ -x - 30 & 3w - 8 \end{bmatrix}$$

5. BC

6. $2D$

$$\begin{bmatrix} -10y & wy \\ 140 & 70 - 6w \\ 108 & 77 - 2w \end{bmatrix}$$

$$\begin{bmatrix} -4 & 0 & 26 \\ 2z & -18 & 12 \\ -12 & 4 & 6 \end{bmatrix}$$

7. Dad's Donuts sell the following types of donuts on Saturday morning: 191 plain for \$0.45, 122 jelly for \$0.55, 98 glazed for \$0.50, and 69 specialty for \$0.85. Write a row matrix to represent the cost of the donuts and a column matrix to represent how many they sold. Then find the total sales for the day using matrix operations.

$$[\$0.45 \quad \$0.55 \quad \$0.50 \quad \$0.85] \begin{bmatrix} 191 \\ 122 \\ 98 \\ 69 \end{bmatrix} = [\$6860.70]$$

8. Birth rates and age specific survival rates for 3-month periods are summarized in the following table.

Age (months)	Birth Rate	Survival Rate
0-3	0	0.6
3-6	0.3	0.9
6-9	0.8	0.9
9-12	0.7	0.8
12-15	0.4	0.6
15-18	0	0

Suppose the original female rat population is 42 animals with the age distribution shown in the following table.

Age (months)	0-3	3-6	6-9	9-12	12-15	15-18
Number	15	9	13	5	0	0

Find the population total and distribution after 3 months (1 cycle) for the following initial populations.

a.) $[35 \ 0 \ 0 \ 0 \ 0 \ 0]$

b.) $[5 \ 5 \ 5 \ 5 \ 5 \ 5]$

21

$[11 \ 3 \ 4.5 \ 4.5 \ 4 \ 3 \ 0]$

9. The students at Central High are planning to hire a band for the prom. Their choices are bands A, B, and C. They survey the Sophomore, Junior, and Senior classes and find the following percentages of students (regardless of sex) prefer the bands,

	10 th	11 th	12 th
A	20%	35%	40%
B	30%	30%	25%
C	50%	35%	35%

The student population by class and sex is:

	Male	Female
10 th	235	225
11 th	205	215
12 th	175	190

Use matrix multiplication to find:

a.) The number of males and females who prefer each band.

$$\begin{bmatrix} 189 & 196 \\ 176 & 180 \\ 251 & 254 \end{bmatrix}$$

b.) The total number of students who prefer each band.

$A-385; B-356, C-505$

10. The characteristics of the female population of a herd of small mammals are shown in the following table.

Age groups (months)

	0-4	4-8	8-12	12-16	16-20	20-24
Birth Rate	0	0.5	1.1	0.9	0.4	0
Survival Rate	0.6	0.8	0.9	0.8	0.6	0

Suppose the initial female population for the herd is given by $P_0 = [22 \ 22 \ 18 \ 20 \ 7 \ 2]$

a.) What is the expected lifespan of this mammal? **20-24 months**

b.) Construct the Leslie matrix for this population.

$$\begin{matrix}
 0 & 0.6 & 0 & 0 & 0 & 0 \\
 0.5 & 0 & 0.8 & 0 & 0 & 0 \\
 1.1 & 0 & 0 & 0.9 & 0 & 0 \\
 0.9 & 0 & 0 & 0 & 0.8 & 0 \\
 0.4 & 0 & 0 & 0 & 0 & 0.6 \\
 0 & 0 & 0 & 0 & 0 & 0
 \end{matrix}$$

c.) Determine the long-term growth rate for the herd. **9.86%**

d.) How many mammals will there be after 4 years? **308**

Outcome E and F: Graph Theory

1. Describe the route that is used for each of the following:

a.) Euler Circuit **Path that uses each edge of a graph exactly once and ends at the starting vertex**

b.) Euler Path **Path that uses each edge of a graph exactly once and ends at a different vertex from the starting vertex**

c.) Hamiltonian Path **Path that uses each vertex of a graph exactly once and ends at the starting vertex**

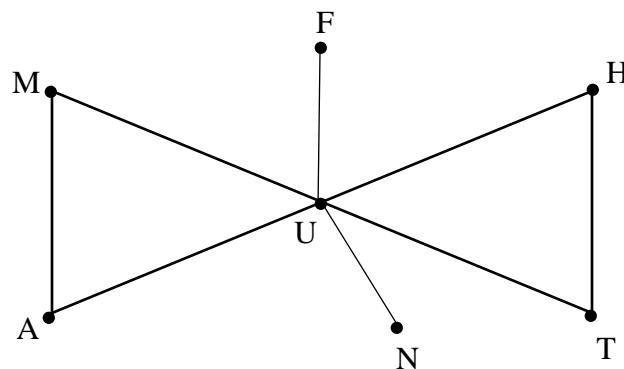
d.) Hamiltonian Path **Path that uses each vertex of a graph exactly once and ends at a different vertex from the starting vertex**

2. Use the picture to the right.

What is the valence of each of the following:

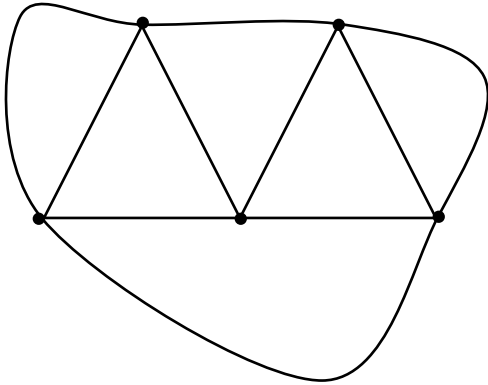
a.) H **2**

b.) F **1**



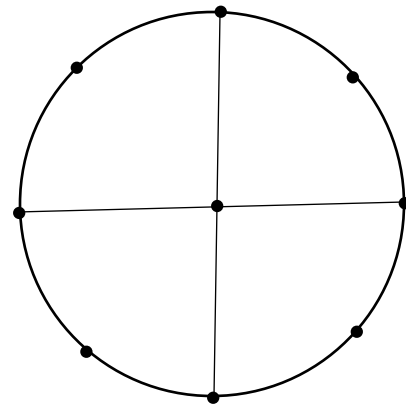
3. State whether each graph has an Euler circuit, Euler path, or neither. Explain why.

a.)



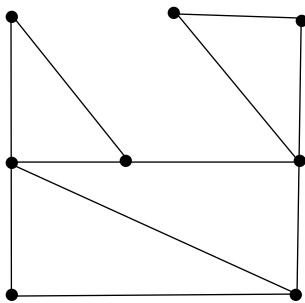
Euler Circuit

b.)



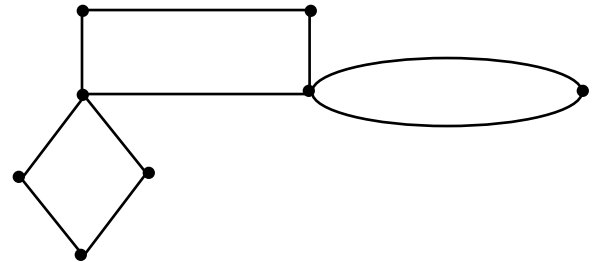
Neither

c.)



Euler Path

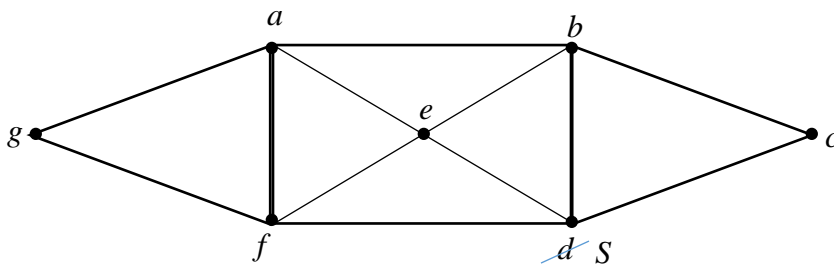
d.)



Euler Circuit

4. Sally began using the Euler circuit algorithm to find the Euler circuit for the following graph. She started at vertex d and labeled it S . The first circuit she found was S, e, f, a, b, c, S . Using Sally's start, continue the algorithm and find an Euler circuit for the graph.

Sample answers: b, e, a, g, f, S OR f, g, a, e, b, S

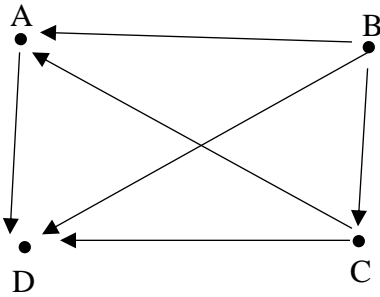


5. Draw a graph with 6 vertices and 12 edges as an Euler Circuit.

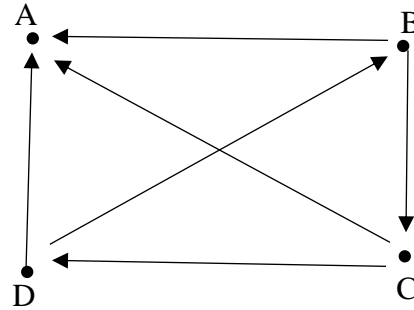
Answers may vary

6. Find all the directed Hamiltonian paths for each of the following tournaments:

a.)

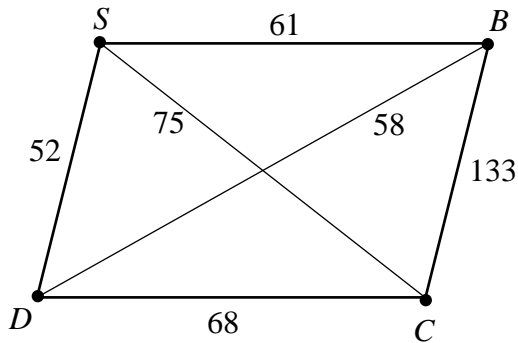


b.)



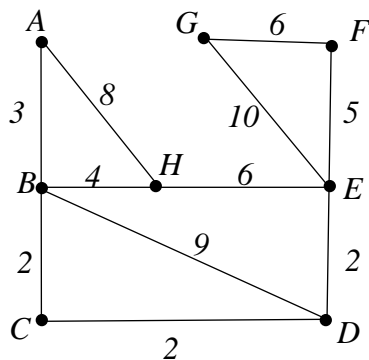
ANSWERS MAY VARY

7. Using the nearest-neighbor algorithm, what is the length of the route which begins and ends at S, visiting each other city exactly once? Does it produce the optimal solution?

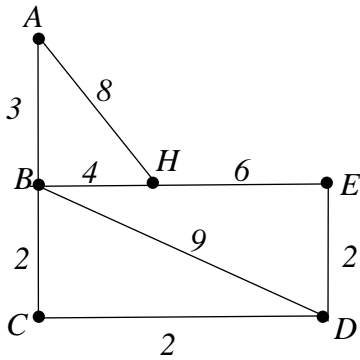


318: No because if you go SDCBS it would be shorter

8. Use the shortest path algorithm to find the shortest route from A to F. 14



9. Use the Traveling Salesman method to determine the minimum distance for a business trip leaving from H. What is the minimum distance? **Sample: HABCDEH: 23**



10. Below is a graph representing the tasks necessary to build a new car with the accompanying times in hours. What is the length of the critical path? **17 HOURS**

