HONORS PRECALC **NAME**_____

SEMESTER 2 FINAL EXAM REVIEW (Units 4, 5, and 6) **PERIOD**_____

UNIT 4

Simplify each expression.

1) $(\sec\theta - \tan\theta)(1 + \tan\theta)$

2) $\frac{\cos\theta}{\sin^2\theta - 1}$

3) $\frac{1+\tan\theta}{1+\cot\theta}$

4) $\frac{\cos 2\theta}{\cos\theta - \sin\theta}$

5) $\frac{\sec^2 x}{\sec^2 x - 1}$

Verify each identity.

6) $\cos x \cot x + \sin x = \csc x$

7) $\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = 2\cot x \csc x$

8) $\frac{1+\cos 2\theta}{2\cos\theta} = \cos\theta$

Use the given information to find ALL possible values of the given trigonometric function. If possible, find the exact value, otherwise round to the nearest hundredth.

9) $\cos\theta = \frac{1}{4}$, find $\tan\theta$	**10)** $\sin\theta = 0.5$, find $\tan\theta$	**11)** $\csc\theta = 2.92$, find $\cot\theta$

Solve each equation for x, $0 \le x < 2\pi$. **Express solutions in radians. Also, find the general solution.**

12) $\cos x = 3\cos x - 2$ Solution if , $0 \le x < 2\pi$: General solution:	**13)** $4\cos^2 x - 1 = 0$ Solution if , $0 \le x < 2\pi$: General solution:
14) $3\cos x + \cos 2x = 0$ Solution if , $0 \le x < 2\pi$: General solution:	**15)** $\cos^2 x - 3\sin x = 3$ Solution if , $0 \le x < 2\pi$: General solution:
16) $\sin 2x - \cos x = 0$ Solution if , $0 \le x < 2\pi$: General solution:	**17)** $2\sec^2 x + \tan^2 x - 3 = 0$ Solution if , $0 \le x < 2\pi$: General solution:

Write each expression as a single trigonometric function.

18) $\sin 75° \cos 20° + \cos 75° \sin 20°$	**19)** $\cos 30° \cos 20° - \sin 30° \sin 20°$

Use a sum or difference identity to find the exact value of each of the following:

20) $\tan \frac{7\pi}{12}$	**21)** $\sin 75°$	**22)** $\cos 105°$

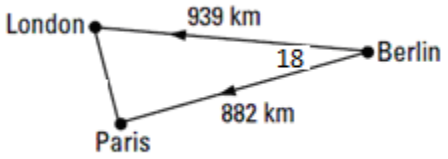
Use an appropriate half-angle identity to find the exact value of each of the following:

23) $\cos 165°$	**24)** $\sin 112.5°$	**25)** $\tan\left(\frac{5\pi}{12}\right)$

26) Find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$ if $\sin\theta = -\frac{2}{5}$, and $\frac{3\pi}{2} < \theta < 2\pi$.

Solve each of the following:

27) Two airplanes leave Berlin, one heading straight for London and the other straight for Paris. The formed is $18°$. Estimate the distance from London to Paris.



28) Bob is on one side of a 260-foot-wide canyon and Sue is on the other. Bob and Sue both see a bear at an angle of depression of 50°. How far are they from the bear? Also, how tall is the canyon?

29) Two ships leave port at the same time. The first ship sails on a bearing of $N\ 40°\ E$ at 18 knots (nautical miles per hour) and the second sails on a bearing of $S\ 60°\ E$ at 26 knots. How far apart are the ships after an hour and a half?

Solve ΔABC given the following:

30) $A = 39.4°,\ b = 12,\ c = 14$

31) $a = 41,\ A = 39°, and\ B = 20°$

32) $A = 51°, a = 40\ and\ c = 50$

33) $A = 65°,\ a = 10, b = 8$

34) In $\Delta ABC, a = 8, b = 11, and\ c = 14$. Find angle C.

UNIT 5

Find the magnitude and direction of the vectors. The direction may be rounded to the nearest hundredth.

1) $\vec{v} =< 4, 5 >$	**2)** $\vec{v} =< -8, -3 >$

Use the following vectors to complete the operations listed below. $\vec{u} =< 7, -2 >, \vec{v} =< 4, -6 >, \vec{w} =< -3, -2 >$

3) $2\vec{w} + \vec{v}$	**4)** $\vec{u} - \vec{v}$

Find the component from of the vectors using the given information. Exact answers only.

5) $\|v\| = 100, \theta = 45°$	**6)** $\|v\| = 20, \theta = 60°$

7) A) Express the vector with an initial point of (13, 4) and a terminal point of (6,-7) in component form.

B) Find the magnitude and direction of the vector found in A.

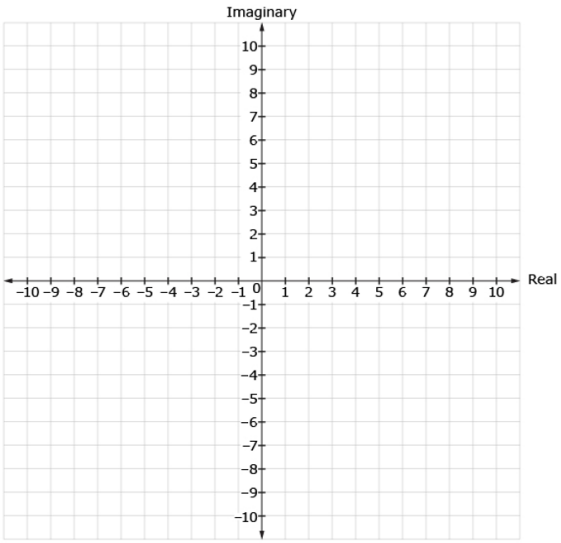
8) A boat is traveling south at 46 mph without the current. The current in the water is traveling at 6 mph $\boldsymbol{S}\ 33°\ \boldsymbol{W}$.

a) Express the velocity of the current in component form.

b) Express the velocity of the boat excluding the current as a vector in component form.

c) Find the true velocity of the boat as a vector.

9) Graph the following complex number in polar form as a vector on the complex plane:
$z = \sqrt{21}(\cos 210° + i \sin 210°)$



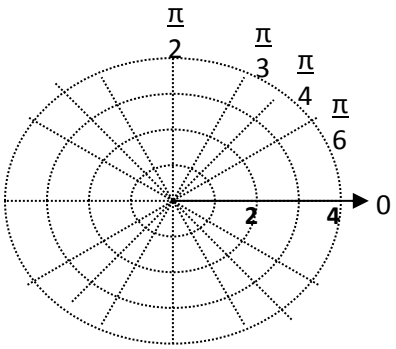
Find the following complex quotients using the complex conjugate.
Confirm your answer with your graphing calculator in *a + bi* mode. Your final answer must be in *a + bi* form!

10) $\frac{4+3i}{2-i}$	**11)** $\frac{3-5i}{1+4i}$

Let $z_1 = 16(\cos 120° + i \sin 120°)$ and $z_2 = 2(\cos 50° + i \sin 50°)$. All answers should be in polar form.

12) What is the absolute value (or **modulus**) of z_1?	**13)** What is the **argument** of z_2?
14) What is the product $z_1 z_2$?	**15)** What is the quotient $\frac{z_1}{z_2}$?
16) What is $(z_2)^2$? $(z_2)^3$? $(z_2)^4$? Describe the pattern you are seeing with these three answers.	**17)** Describe the pattern you see with the three answers found in #16.

18) Graph the polar coordinates $\left(2,\frac{2\pi}{3}\right)$, $(-3,-135°)$, $\left(-1,\frac{11\pi}{6}\right)$, $(4,-270°)$ on the graph provided.



19) Write the ordered pair (-3,-1) as polar coordinates.

20) Write the polar coordinates $\left(-4,\frac{5\pi}{3}\right)$ in three other ways.

21) Write the equation $x^2+2y^2=5$ in polar form.

22) Write the equation $2\sin\theta = r$ in rectangular form.

23) Let $z_1 = 2-2i$; $z_2 = 1+2i$; and $z_3 = -8i$

a) Find $z_1 z_2$ (you may use cis notation)

b) Find $(z_2)^4$ (you may use cis notation)

Describe the following polar graphs and sketch a sample of what it looks like. Describe the form of the equation that would be used to graph each.

24) Cardiod	**25)** limacon	**26)** rose curve

For questions 27 and 28, write the equation into standard form for the type of conic.

27) $-y^2 + x + 10y - 26 = 0$	**28)** $x^2 + y^2 + 4x - 2y - 14 = 0$

For numbers 29-32, state the information for each conic in standard form.

29) $(x-4)^2 + (y+1)^2 = 400$ **Center:_____** **Radius:_____**	**30)** $\frac{(x+3)^2}{81} + \frac{y^2}{16} = 1$ **Center: _____** **Length of Major Axis:_____** **Length of Minor Axis:_____** **Vertices:_____** **Co-vertices:_____** **Foci:_____**
31) $\frac{(y+1)^2}{100} - (x-9)^2 = 1$ **Center: _____** **Length of Transverse Axis:_____** **Length of Conjugate Axis:_____** **Vertices:_____** **Foci:_____** **Equations of Asymptotes:_____**	**32)** $x - 2 = \frac{1}{4}(y+1)^2$ **Vertex:_____** **Direction of Opening:_____** **Equation of Axis of Symmetry:_____** **Equation of Directrix:_____** **Focus:_____**

For problems 33-34, write the equation of the requested conic section in standard form for conics.

33) *Hyperbola* Vertices: (1, 2), (-15, 2) Length of conjugate axis: 24	**34)** *Parabola* Focus: (-2, 9) Directrix: $y = 7$

35) A narrow arch supporting a stone bridge is in the shape of half of an ellipse and 24 meters long and 8 meters high. A person standing at one location throws a rubber ball against the arch. No matter what direction the ball is thrown, it always bounces off the arch once and strikes the same point on the ground. How far apart are the person throwing the ball and point on the ground at which the ball first strikes?

UNIT 6

Use the following matrices for problems 1- 12. If the operation cannot be performed, state the reason why.

$$A=\begin{bmatrix}1 & 4\\ 0 & -2\end{bmatrix} \qquad B=\begin{bmatrix}9 & -3\\ -12 & 4\end{bmatrix} \qquad C=\begin{bmatrix}7 & 6\end{bmatrix} \qquad D=\begin{bmatrix}9 & 1 & 7\\ 2 & 0 & 2\\ 5 & 1 & 6\end{bmatrix} \qquad E=\begin{bmatrix}7\\ -2\\ -3\end{bmatrix}$$

1) $A+B$	**2)** $B-3A$	**3)** $B+2C$
4) AB	**5)** CB	**6)** DE
7) $\lvert A\rvert$	**8)** $\lvert B\rvert$	**9)** $\lvert D\rvert$
10) A^{-1}	**11)** B^{-1}	**12)** C^{-1}

13) For what value(s) of x does this matrix not have an inverse? $\begin{bmatrix}2x-3 & x-1\\ x+6 & x+2\end{bmatrix}$

14) Solve this matrix equation for x and y: $\begin{bmatrix}x^2 & 2\\ 4y & x+1\end{bmatrix}+\begin{bmatrix}-2x & 3y\\ -7 & x^2\end{bmatrix}=\begin{bmatrix}3 & 11\\ 5 & 1\end{bmatrix}$

15) Akena bought several bottles of nail polish and some nail files at Sally's Beauty Supply. The bottles of polish were $6 each and the files were $2 each. Before taxes, her total was $34 for the 11 items. Write this scenario as a matrix equation and find how many of each item Akena purchased.

16) ABC Totes makes large canvas bags and small zipper pouches. The bags take 14 minutes each to cut and 10 minutes to sew. The pouches take 5 minutes each to cut and 8 minutes to sew. A local gift shop has just ordered 26 bags and 30 pouches. If cutting costs $1 per minute and sewing costs $1.50 per minute, what is the total cost of this order?

17) Solve the following system of equations using matrices:

$$\begin{cases} x + 2y + 3z = 4 \\ 4x - 2y = 7 \\ z + 10 = 2x + 3y \end{cases}$$

18) Find the area of the triangle with vertices (-4, 2), (2, -6) and (4, 8)

19) Given the quadrilateral with vertices A(-5, 3), B(-1, -6), C(7, -2) and D(6, 5), find each of the following transformation using matrices.

a) Translate 3 units right and 5 units down.

b) Dilate vertically by a factor of 3 and horizontally by a factor of $\frac{1}{2}$.

20) By what angle does this matrix rotate a point clockwise about the origin? $(0 \leq \theta < 360°)$ $\begin{bmatrix} -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$

21) The point (x, y) was rotated clockwise about the origin 150° to the point $\left(\frac{-3\sqrt{3}-1}{2}, \frac{-3+\sqrt{3}}{2}\right)$. What was the original point (x, y)?