

## Precalculus

## Semester 2 Review Units 4, 5, and 6

NAME: \_\_\_\_\_

Period: \_\_\_\_\_

## UNIT 4

Simplify each expression.

1)  $(\sec \theta - \tan \theta)(1 + \tan \theta)$

2)  $\frac{\cos \theta}{\sin^2 \theta - 1}$

3)  $\frac{1 + \tan \theta}{1 + \cot \theta}$

4)  $\frac{\cos 2\theta}{\cos \theta - \sin \theta}$

5)  $\frac{\sec^2 x}{\sec^2 x - 1}$

Verify each identity.

6)  $\cos x \cot x + \sin x = \csc x$

7)  $\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = 2 \cot x \csc x$

8)  $\frac{1 + \cos 2\theta}{2 \cos \theta} = \cos \theta$

Use the given information to find ALL possible values of the given trigonometric function. If possible, find the exact value, otherwise round to the nearest hundredth.

<b>9)</b> $\cos \theta = \frac{1}{4}$ , find $\tan \theta$	<b>10)</b> $\sin \theta = 0.5$ , find $\tan \theta$	<b>11)</b> $\csc \theta = 2.92$ , find $\cot \theta$
---	--	---

Solve each equation for  $x$ ,  $0 \leq x < 2\pi$ . Express solutions in radians.

<b>12)</b> $\cos x = 3 \cos x - 2$	<b>13)</b> $4 \cos^2 x - 1 = 0$
<b>14)</b> $3 \cos x + \cos 2x = 0$	<b>15)</b> $\cos^2 x - 3 \sin x = 3$
<b>16)</b> $\sin 2x - \cos x = 0$	<b>17)</b> $2 \sec^2 x + \tan^2 x - 3 = 0$

Write each expression as a single trigonometric function.

18)  $\sin 75^\circ \cos 20^\circ + \cos 75^\circ \sin 20^\circ$

19)  $\cos 30^\circ \cos 20^\circ - \sin 30^\circ \sin 20^\circ$

Use a sum or difference identity to find the exact value of each of the following:

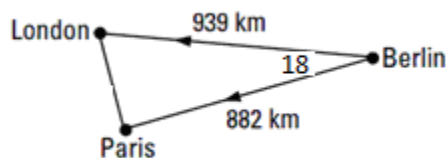
20)  $\tan \frac{7\pi}{12}$

21)  $\sin 75^\circ$

22)  $\cos 105^\circ$

23) Find  $\sin 2\theta$ ,  $\cos 2\theta$ , and  $\tan 2\theta$  if  $\sin \theta = -\frac{2}{5}$ , and  $\frac{3\pi}{2} < \theta < 2\pi$ .

24) Two airplanes leave Berlin, one heading straight for London and the other straight for Paris. The angle formed is 18 degrees. Estimate the distance from London to Paris.



**25)** Bob is on one side of a 260-foot-wide canyon and Sue is on the other. Bob and Sue both see a bear at an angle of depression of 50 degrees. How far are they from the bear?

**26)** Two ships leave port at the same time. The first ship sails on a bearing of  $N 40^\circ E$  at 18 knots (nautical miles per hour) and the second sails on a bearing of  $S 60^\circ E$  at 26 knots. How far apart are the ships after an hour and a half?

Solve  $\triangle ABC$  given the following:

**27)**  $A = 39.4^\circ$ ,  $b = 12$ ,  $c = 14$

**28)**  $a = 41$ ,  $A = 39^\circ$ , and  $B = 20^\circ$

**29)**  $A = 51^\circ$ ,  $a = 40$  and  $c = 50$

**30)**  $A = 65^\circ$ ,  $a = 10$ ,  $b = 8$

**31)** In  $\triangle ABC$ ,  $a = 8$ ,  $b = 11$ , and  $c = 14$ . Find angle  $C$ .

**UNIT 4**

Write the equation of each parabola:

<p><b>1)</b> Vertex: (1, -1)</p> <p>Focus: <math>(1, -\frac{3}{2})</math></p>	<p><b>2)</b> Vertex: (-3, -8)</p> <p>Focus: (-2, -8)</p>
<p><b>3)</b> Vertex: (-8, -2)</p> <p>Focus: <math>(-8, -\frac{7}{4})</math></p> <p><b>A)</b> <math>y = (x - 8)^2 - 2</math>  <b>B)</b> <math>y = -(x + 8)^2 - 2</math>  <b>C)</b> <math>y = (x + 8)^2 - 2</math>  <b>D)</b> <math>y = -(x - 2)^2 + 8</math></p>	<p><b>4)</b> Vertex: (1, 6)</p> <p>Focus: <math>(\frac{1}{2}, 6)</math></p> <p><b>A)</b> <math>x = -(y + 5)^2 + 1</math>  <b>B)</b> <math>x = -\frac{1}{2}(y - 6)^2 + 1</math>  <b>C)</b> <math>x = -\frac{1}{2}(y + 6)^2 + 1</math>  <b>D)</b> <math>x = \frac{1}{2}(y - 6)^2 + 1</math></p>

Identify the Vertex, Focus, and Directrix

<p><b>5)</b> <math>y = x^2 - 18x + 74</math></p>	<p><b>6)</b> <math>x = -y^2 + 4y - 11</math></p>
<p><b>7)</b> <math>y = x^2 - 16x - 63</math></p> <p><b>A) Vertex: (-8, 1)</b>      <b>B) Vertex: (-3, -2)</b>  <b>Focus: <math>(-8, \frac{3}{4})</math></b>      <b>Focus: (-4, -2)</b>  <b>Directrix: <math>y = \frac{5}{4}</math></b>      <b>Directrix: <math>x = -2</math></b></p> <p><b>C) Vertex: (-3, -2)</b>      <b>D) Vertex: (3, 2)</b>  <b>Focus: (-2, -2)</b>      <b>Focus: (3, 3)</b>  <b>Directrix: <math>x = -4</math></b>      <b>Directrix: <math>y = 1</math></b></p>	<p><b>8)</b> <math>x = -y^2 + 5</math></p> <p><b>A) Vertex: (-1, -4)</b>      <b>B) Vertex: (0, 5)</b>  <b>Focus: <math>(-\frac{5}{4}, -4)</math></b>      <b>Focus: <math>(\frac{1}{4}, 5)</math></b>  <b>Directrix: <math>x = -\frac{3}{4}</math></b>      <b>Directrix: <math>x = -\frac{1}{4}</math></b></p> <p><b>C) Vertex: (5, 0)</b>      <b>D) Vertex: (1, -6)</b>  <b>Focus: <math>(\frac{19}{4}, 0)</math></b>      <b>Focus: <math>(1, -\frac{23}{4})</math></b>  <b>Directrix: <math>x = \frac{21}{4}</math></b>      <b>Directrix: <math>y = -\frac{25}{4}</math></b></p>

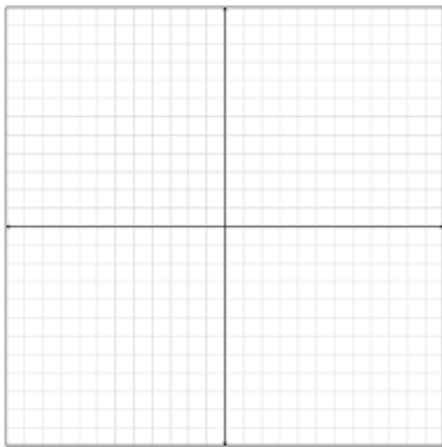
9) The center of an ellipse is the point (5,2). One focus of the ellipse is at (8,2), and one vertex of the ellipse is at (10,2).

a) What are the coordinates of the other focus and the other vertex? Explain your answer.

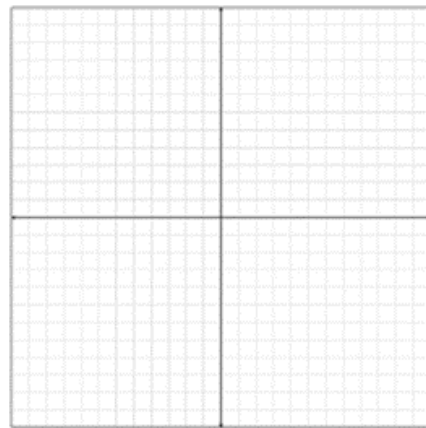
b) What is the equation of the ellipse?

Sketch the graph of each ellipse or hyperbola. If it is an ellipse, label the coordinates of the center, the foci, and the endpoints of the major and minor axes. If it is a hyperbola, label the coordinates of the center, the foci, and the vertices, write the equations of the asymptotes.

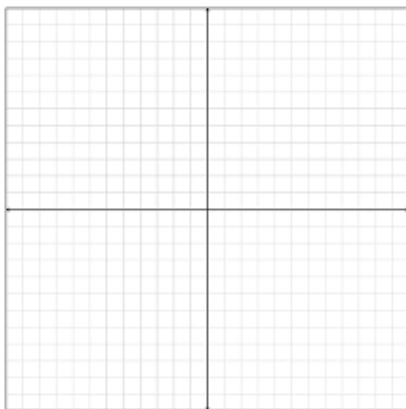
10)  $\frac{x^2}{144} + \frac{y^2}{169} = 1$



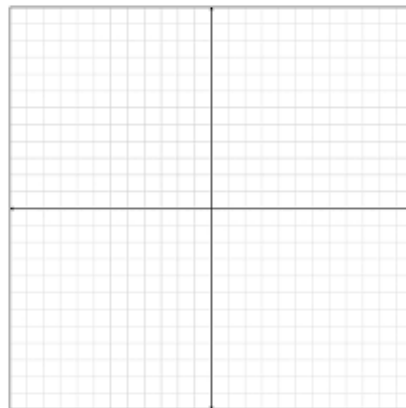
11)  $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{4} = 1$



12)  $\frac{x^2}{25} - \frac{y^2}{16} = 1$

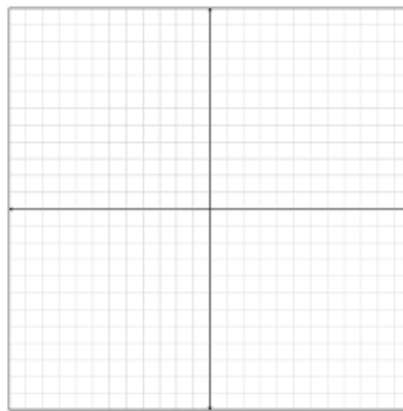


13)  $\frac{(y-2)^2}{16} - \frac{(x+3)^2}{9} = 1$



**14)** Sketch the graph for the following ellipse. Label the coordinates of the center, the foci, and the endpoints of the major and minor axes.

$$4x^2 + 3y^2 + 8x - 6y - 5 = 0$$



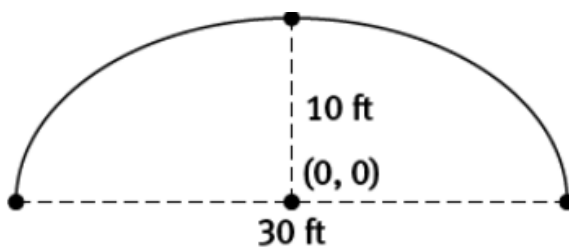
**15)** The center of a hyperbola is the point  $(1, -3)$ . One focus of the hyperbola is at  $(-3, -3)$ , and one vertex of the hyperbola is at  $(-2, -3)$ .

**a)** What are the coordinates of the other focus and vertex?

**b)** What is the equation of the hyperbola?

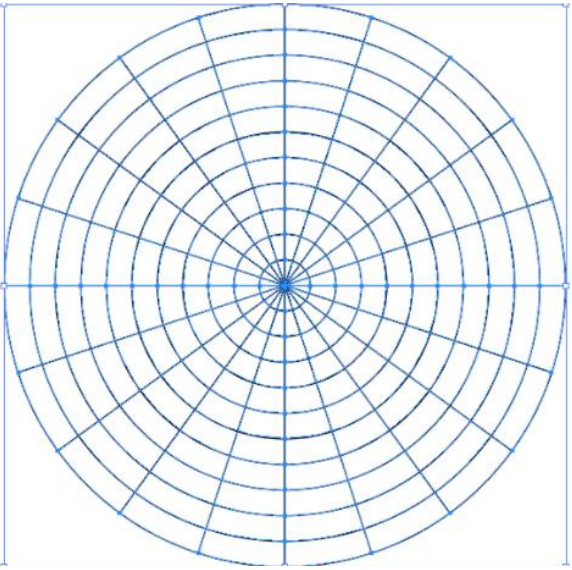
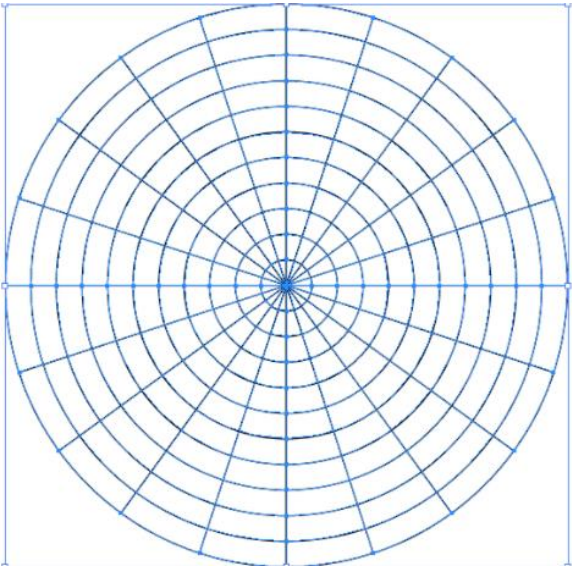
**16)** The bottom surface of a bridge forms a semi-elliptical arch over a river. The width of the arch is 30ft, and the height is 10ft.

Write the equation of the ellipse that models the arch. Let  $(0, 0)$  on a coordinate plane in units of feet represent the center of the ellipse and the x-axis represent the water line.



<p><b>17) Choose the rectangular form of the polar equation given: <math>r = -2 \cos\theta + 4 \sin\theta</math></b></p> <p>a) <math>(x - 1)^2 + (y + 2)^2 = 5</math>  b) <math>(x + 1)^2 + (y - 2)^2 = 5</math>  c) <math>(x - 2)^2 + (y + 4)^2 = 20</math>  d) <math>(x + 2)^2 + (y - 4)^2 = 20</math></p>	<p><b>18) Find the rectangular form of the polar equation given. <math>r = 4 \cos\theta - 2 \sin\theta</math></b></p>
<p><b>19) Convert the polar coordinates of <math>(3, 3\sqrt{3})</math> to rectangular coordinates.</b></p>	<p><b>20) Convert the rectangular coordinates of <math>(6, 150^\circ)</math> to polar coordinates.</b></p>

Graph the following polar equations. Identify whether it is a rose curve, cardioid, or limacon.

<p><b>21) <math>r = 4 + 2 \cos(\theta)</math></b></p> 	<p><b>22) <math>r = 3 - 3 \sin(\theta)</math></b></p> 
--	---

Convert the equation from polar to rectangular form.

<p><b>23) <math>r = 6 \cos\theta + 2 \sin\theta</math></b></p>	<p><b>24) <math>r = -8 \cos\theta + 4 \sin\theta</math></b></p>
--	---



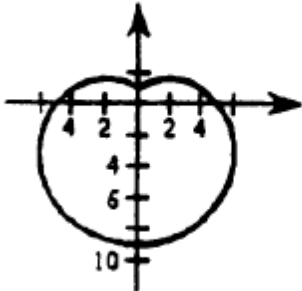
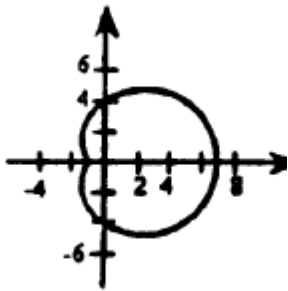
Convert the following coordinates from rectangular to polar form. Answers should be exact value.

25) $(4, 150^\circ)$	26) $(1, 270^\circ)$
27) $(5, 300^\circ)$	28) $(2, 45^\circ)$

Convert the following coordinates from rectangular form to polar form where  $r > 0$  and  $0^\circ \leq \theta \leq 360^\circ$ .

29) $(2\sqrt{2}, -2\sqrt{2})$	30) $(-5\sqrt{3}, 5)$	31) $(-1, -\sqrt{3})$
-------------------------------	-----------------------	-----------------------

Find the equation for the graph.

<p>32)</p>  <p>a. <math>r = 5 + 4 \sin \theta</math>          b. <math>r = 4 - 5 \sin \theta</math>          c. <math>r = 4 + 5 \sin \theta</math>          d. <math>r = 5 - 4 \sin \theta</math>          e. None of the above</p>	<p>33)</p>  <p>a. <math>r = 4 - 3 \cos \theta</math>          b. <math>r^2 = 9 \cos \theta</math>          c. <math>r = 3 \cos 3\theta</math>          d. <math>r = 4 + 3 \cos \theta</math>          e. None of the above</p>
--	--

Resolve the vector into its components. Give exact answers.

<b>34)</b> Magnitude = 8 Direction = $225^\circ$	<b>35)</b> Magnitude = 33 Direction = $120^\circ$
--	---

Find the magnitude and direction angle for the given vector:

<b>36)</b> $\vec{v} = \langle 22, -44 \rangle$	<b>37)</b> $\vec{u} = \langle -7, 7 \rangle$
--	--

Simplify. The answer is to be given in rectangular form  $a + bi$ .

<b>38)</b> $\frac{4-2i}{6+11i}$	<b>39)</b> $\frac{5+3i}{9-2i}$
---------------------------------	--------------------------------

Write in the polar form of a complex number.

<b>40)</b> $-2 + 2i$	<b>41)</b> $8 - 15i$
----------------------	----------------------

## UNIT 6

- 1) A party planner needs to order doughnuts (glazed doughnuts, cake doughnut, jelly filled doughnuts and maple bars) for a brunch they are planning. Donut Den sells doughnuts for \$0.95, \$0.99, \$1.09 and \$1.15 respectively. Crispy Kreme sells doughnuts for \$ 0.99, \$0.99, \$1.15, and \$1.20 respectively. Dunkin sells doughnuts for \$0.90, \$0.95, \$1.25, and \$1.20 respectively.

- 2) **Represent the data in a single matrix. Label the rows and column.**

The party planner decides to purchase 125 glazed doughnuts, 90 cake doughnuts, 60 Jelly filled and 100 maple bars. What is the total price from each doughnut shop? Which doughnut shop would be the most economical place to purchase doughnuts? Justify/explain your answer.

Find the inverse of the following matrices. Show your work

3)  $\begin{bmatrix} -8 & 24 \\ -1 & 6 \end{bmatrix}$

4)  $\begin{bmatrix} -1 & 0 \\ -3 & 6 \end{bmatrix}$

Solve each of the following using matrices. Show your matrix equation.

- 5) The admission fee at a small fair is \$1.50 for children and \$4.00 for adults. On a certain day, 2200 people enter the fair and \$5050 is collected. How many children and how many adults attended?

6) A farmer has chickens and pigs. There were a total of 60 heads and 200 feet. How many chickens, and how many pigs did the farmer have?

7) A company sells three types of movie gift baskets. A basic basket with 2 movie passes and 1 package of microwave popcorn cost \$15.50. A medium basket with 2 movie passes, 2 packages of popcorn, and 1 DVD costs \$37. A super basket with 4 movie passes, 3 packages of popcorn, and 2 DVDs cost \$72.50. Find the cost of each item in the gift baskets.

Solve for  $x$  and  $y$ . Find the possible solutions.

8)

$$\begin{bmatrix} 4x & 17 \\ y+2 & -42 \end{bmatrix} = \begin{bmatrix} -24 & 17 \\ 15 & -42 \end{bmatrix}$$

9)

$$\begin{bmatrix} 4x & y^2 \\ y^2 & 7x \end{bmatrix} + \begin{bmatrix} -24 & -4y \\ -2y-3 & -42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

10) A plot of land is marked on the coordinate plane by the following points  $(2, 2)$ ,  $(-2, 1)$ ,  $(-3, -2)$ ,  $(1, -1)$ .

a) Using matrices create a horizontal stretch by a factor of 3.

b) Using matrices create vertical stretch by a factor of 2.

c) Find the coordinates of the point  $(5, 8)$  after a rotation of  $67^\circ$  clockwise about the origin. Round to the nearest hundredth.

d) Find the coordinates of the point  $(-2, 6)$  after a rotation of  $83^\circ$  clockwise about the origin. Round to the nearest hundredth.

11) Find the coordinates of the point  $(-2, 6)$  after a rotation of  $83^\circ$  clockwise about the origin. Round to the nearest hundredth.

Two endpoints of a line segment are listed as  $(-3, 4)$  and  $(5, 3)$ . The line needs to be shifted 4 units right and 6 units up. Find the new coordinates of the transformation. You need to be able to express the transformation as all three of the following

a) The sum of matrices

b) The difference of matrices

c) Incorporating scalar multiplication with either sum or difference