Name ___________________________

AP Calculus BC – 1st Semester Final Review

In problems #1-6, evaluate the limit, if it exists.

1. $\lim_{x \to -2} \frac{x^2+3x+2}{x+2}$

2. $\lim_{h \to 0} \frac{(3+h)^3-27}{h}$

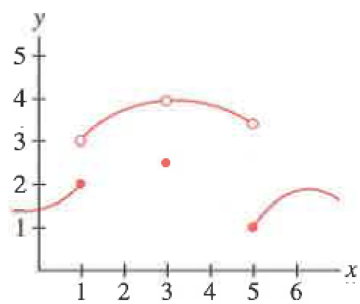
3. $\lim_{x \to 8} \frac{\sqrt{x-4}-2}{x-8}$

4. $\lim_{x \to 2} \frac{x^3-8}{x-2}$

5. $\lim_{x \to \infty} \frac{3x^2+20x}{2x^4+3x^3-29}$

6. $\lim_{x \to -\infty} \frac{\sqrt{9x^4+3x+2}}{4x^2+1}$

The graph below is the graph of $y = f(x)$. Use the graph to answer questions #7-9



7. At what value of *x* is the graph above "discontinuous" but the limit still "exists."

8. $\lim_{x \to 1^-} f(x) =$

9. $\lim_{x \to 5^-} f(x) =$

Differentiate.

10. $f(x) = \frac{e^x}{x^2+1}$

11. $h(t) = \frac{t^2-1}{t-1}$

12. $f(x) = \sqrt{x^2+9}$

Find y'

13. $y = \left(\frac{x+1}{x-1}\right)^4$

14. $y = \sin(x^2)\cos(x^2)$

15. $y = 7^{4x-x^2}$

16. $s(t) = \ln(8-4t)$

17. $y = e^{(\ln x)^3}$

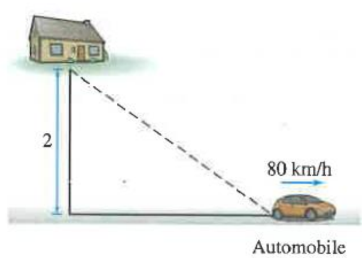
18. $x^2y + 2x^3y = x + y$

19. $xy^2 + x^2y^5 - x^3 = 3$

20. Find the equation of the line tangent to the curve $y = \frac{x^4-4}{x^2-5}$ when $x = 2$.

21. Find an equation of the tangent line to the curve $x^2 + \sin(y) = xy^2 + 1$ at the point $(1, 0)$.

22. A road perpendicular to a highway leads to a farmhouse located 2 km away. An automobile travels past the farmhouse at a speed of 80 km/h. How fast is the distance between the automobile and the farmhouse increasing when the automobile is 6km past the intersection of the highway and the road?



23. A conical tank has height 3m and radius 2m at the top. Water flows in at a rate of 2 m^3/min. How fast is the water level rising when it is 2m?

For questions 24-26, assume that the radius *r* of a sphere is expanding at a rate of 30 cm/min. Determine the given rate.

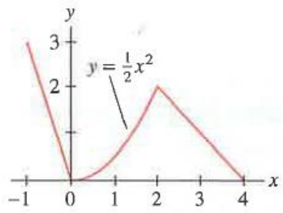
24. Volume with respect to time when $r = 15$ cm.

25. Volume with respect to time at $t = 2$ min, assuming that $r = 0$ when $t = 0$.

26. Surface area with respect to time when $r = 40$ cm.

Determine the values of *x* at which the function is "nondifferentiable."

27.



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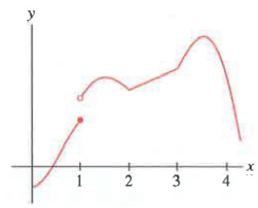


For problems 29 & 30, find the intervals on which the function is increasing or decreasing and determine whether the critical point(s) is a local max or min (or neither).

29. $y = x^3 - 12x^2$

30. $y = x^5 + x^3 + 1$

For problems 31 & 32, determine the intervals on which the function is concave up or down and find the points of inflection.

31. $y = x^3 - 6x^2 + 4$

32. $y = 5x^2 + x^4$

33. Sketch the graph of a function $f(x)$ that satisfies all of the given conditions:

(i) $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$, and

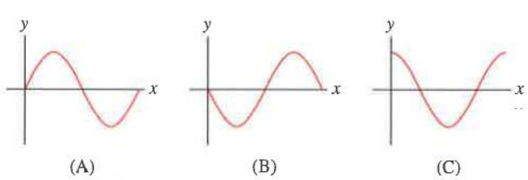
(ii) $f''(x) < 0$ for $|x| > 2$, and $f''(x) > 0$ for $|x| < 2$

34. The height at time *t* (in sec) of a mass, oscillating at the end of a spring, is $s(t) = 300 + 40 \sin t$ cm.

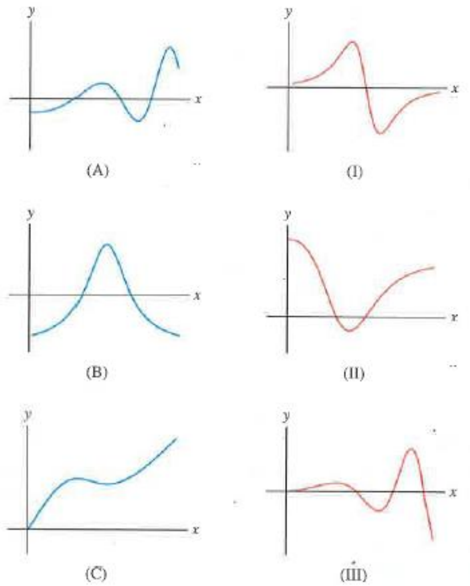
a) Find the velocity and acceleration when $t = \frac{\pi}{3} s$.

b) Find the first time that the mass will be at rest.

35. The graphs below are f, f', f''. Determine which is which.



36. Match functions (A) – (C) with their derivatives (I) – (III) below.



Evaluate the integral.

37. $\int_{-1}^{1}(5u^4 + u^2 - u)du$

38. $\int_{\frac{8}{27}}^{1} \frac{10t^{\frac{4}{3}} - 8t^{\frac{1}{3}}}{t^2} dt$

39. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin\theta \, d\theta$

40. $\int \frac{x^2}{x^3+2} dx$

41. $\int \frac{e^x}{\sqrt{e^x+1}} dx$

42. $\int \frac{4\ln x + 5}{x} dx$

43. $\int \frac{dx}{\sqrt{9-16x^2}}$

44. $\int \frac{tdt}{\sqrt{1-t^4}}$

45. $\int x^{\frac{1}{2}} \cos(x^{\frac{3}{2}}) \, dx$

46. $\int \cos x \cos(\sin x) dx$

47. $\int \cos x \, (3\sin x - 1) \, dx$

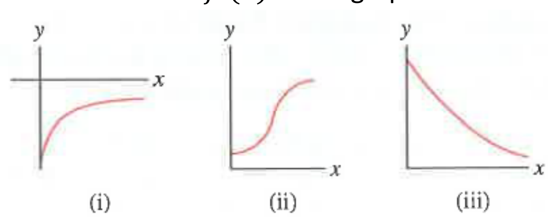
48. $\int \sin(4\theta - 7) \, d\theta$

49. Match the description of $f(x)$ with the graph of the its *derivative* $f'(x)$ in the graphs below.

(a) $f(x)$ is increasing and concave up.

(b) $f(x)$ is decreasing and concave up.

(c) $f(x)$ is increasing and concave down.



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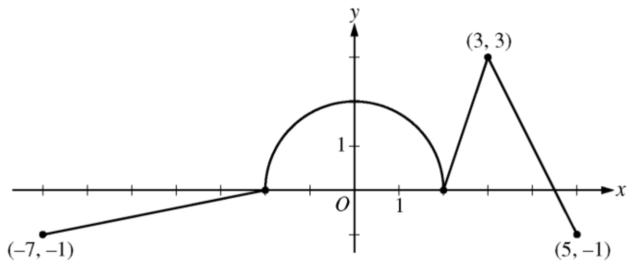


Graph of g'

The graph above is the graph of g'. Answer the following questions and justify your answers.

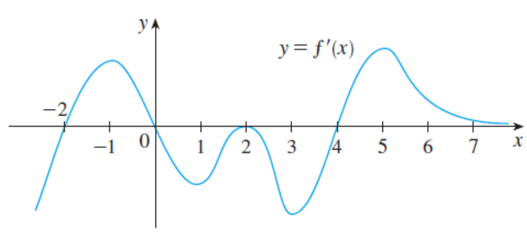
a) On what intervals is $g(x)$ increasing? decreasing?

b) On what intervals is $g(x)$ concave up? Concave down?

c) When does $g(x)$ have a relative maximum? relative minimum?

d) When does $g(x)$ have an inflection point?

51.



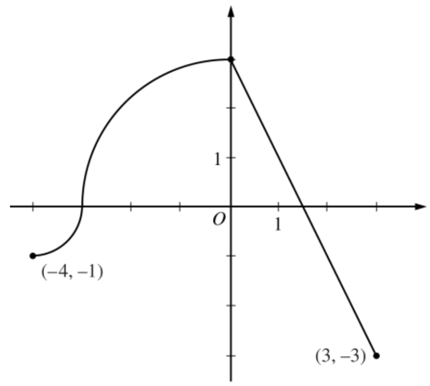
The graph above is the graph of $f'(x)$. Answer the following questions and justify your answers.

a) When does the graph of $f(x)$ have a relative maximum? Relative minimum?

b) On what interval is $f(x)$ both decreasing AND concave up?

c) When does the graph of $f(x)$ have an inflection point?

52.



Graph of f

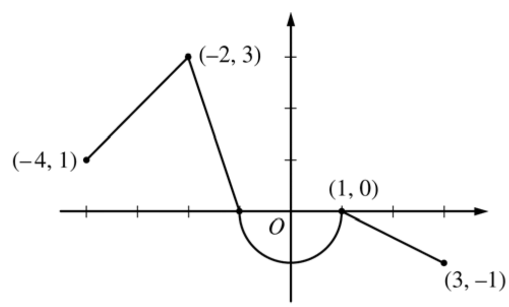
Let f be the continuous function defined on [-4, 3] whose graph, consists of 2 quarter circles and one line segment, as shown in the figure above. Let $g(x) = \int_0^x f(t)dt$.

a) Find $g(-3)$ and $g'(-3)$.

b) Determine the x-coordinate of the point at which g has an absolute maximum on the closed interval $-4 \leq x \leq 3$. Show work that leads to your answer.

c) Find when intervals when the graph of $g(x)$ is increasing and decreasing. Justify your answers.

53.



Graph of f

Let f be the continuous function defined on [-4, 3] whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let $g(x) = \int_1^x f(t)dt$.

a) Find the values of $g(2)$ and $g'(2)$.

b) Find the intervals when the graph of $g(x)$ is increasing and decreasing. Justify your answers.

c) On the closed interval from [-4, 3] find the absolute maximum and absolute minimum of $g(x)$. Show all work to justify your answer.