Name __________________________

AP Calculus AB – 1st Semester Final Review

Part I: Complete the following derivative formulas to see which ones you need to practice!!

Power Rule:

Product Rule:

Quotient Rule:

$\frac{d}{dx}(\sin x) =$

$\frac{d}{dx}(\cos x) =$

$\frac{d}{dx}(\tan x) =$

$\frac{d}{dx}(\csc x) =$

$\frac{d}{dx}(\sec x) =$

$\frac{d}{dx}(\cot x) =$

$\frac{d}{dx}(c) =$

***Recall that all of the above formulas involve x rather than a more complicated function of x. In the event of the latter, **THE CHAIN RULE** requires that you multiply your answer by the derivative of that more complicated function of x.

Remember the different notations used for the first and 2nd derivatives:

1st derivative: y', $\frac{dy}{dx}$, D_x, $f'(x)$, etc.

2nd derivative: y'', $\frac{d^2y}{dx^2}$, $D_x{}^2$, $f''(x)$, etc.

Find the derivative of $y = 3x^2 + 2x + 4$ using the **Definition(s) of the derivative:**

$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ and/or $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Refresh your memory regarding the following theorems:

a. Mean Value Theorem
b. Rolle's Theorem
c. Squeeze Theorem
d. Intermediate Value Theorem
e. Extreme Value Theorem

What are the three requirements for continuity?

1.
2.
3.

Draw a diagram below for each of the following (If impossible, then don't draw it!) :

1. A curve that is continuous but not differentiable at x = a
2. A curve that is neither continuous nor differentiable at x = a
3. A curve that is differentiable but not continuous at x = a
4. A curve that has a limit as x $\rightarrow a^+$ but not as x $\rightarrow a^-$. What would you say about the limit of this curve as x $\rightarrow a$?

Part II: Review problems – do on a separate sheet!

Evaluate the limits.

1. $\lim_{x\to\infty} \frac{2x^2-3x}{8x^2+8}$

2. $\lim_{x\to\infty} \frac{3x^2+20x}{2x^4+3x^3-29}$

3. $\lim_{x\to2} \frac{x^2+x-6}{x^2-x-2}$

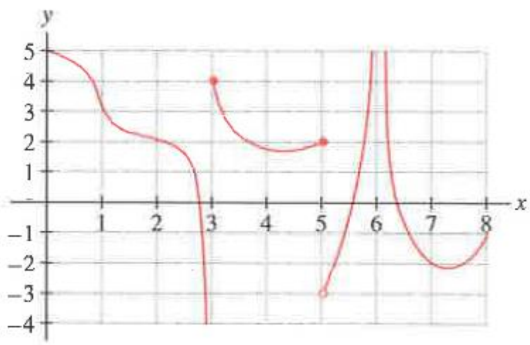
4. $\lim_{x\to1} \frac{\sqrt{x}-1}{x-1}$

In problems 5 & 6, using the definition of continuity, determine whether or not the function given is continuous. If discontinuous, explain why.

5. $f(x) = \begin{cases} x^2, & x \leq 1 \\ 2-x, & x > 1 \end{cases}$

6. $f(x) = \begin{cases} x+1, & x < 1 \\ \frac{1}{x}, & x \geq 1 \end{cases}$

The graph for *f(x)* is shown below. Use the graph of *f(x)* to evaluate the limits.



7. $\lim_{x\to 3^+} f(x)$

8. $\lim_{x\to 5^+} f(x)$

9. $\lim_{x\to 6} f(x)$

10. $\lim_{x\to 5^-} f(x)$

Differentiate.

11. $f(x) = 4x^{\frac{5}{3}} - 3x^{-2} - 12$

12. $f(x) = 6\sqrt{x} + \frac{1}{\sqrt{x}}$

13. $f(x) = \frac{x+4}{x^2+x+1}$

14. $f(x) = x^2(3 + x^{-1})$

15. $f(x) = (x^2 + 9x)^{-2}$

16. $f(x) = \cos^3(12x)$

Find y'.

17. $x^2y + 2x^3y = x + y$

18. $xy^2 + x^2y^5 - x^3 = 3$

Evaluate.

19. $f'(3)$ if $f(x) = \frac{1}{x+10}$

20. $f'(-2)$ if $f(x) = \frac{x}{3x^2+1}$

Find the equation of the line(s) tangent to the graph of the given equation at the given *x*-coordinate or point.

21. $f(x) = \frac{x^4-4}{x^2-5}\,; x = 2$

22. $f(x) = \frac{\sin x}{1+\cos x}; x = \frac{\pi}{3}$

23. $f(x) = (x^4 - x^3 - 1)^{\frac{2}{3}}; x = 1$

24. $xy + x^2y^2 = 5; (2, 1)$

25. The position (in meters) of a particle moving along the y – axis is modeled by the function

$$h(t) = 4t^{-3} + 3t^2.$$

a) Find the particle's velocity and acceleration at time $t = 1$s.

b) Is the particle speeding up, slowing down, or neither at $t = 1\text{s}$? Justify.

For questions 26 and 27, assume that the function given satisfies the conditions for the Mean Value Theorem. Find a point "*c*" that satisfies the conclusion of MVT for the given function and interval.

26. $y = \frac{x}{x+2}; [1, 4]$

27. $y = \cos x - \sin x\,; [0, 2\pi]$

For questions 28 and 29, assume that the function given satisfies the conditions for Rolle's Theorem. Find a point "*c*" that satisfies the conclusion of Rolle's Theorem for the given function and interval.

28. $y = x + x^{-1}; [\frac{1}{2}, 2]$

29. $y = \sin x\,; [\frac{\pi}{4}, \frac{3\pi}{4}]$

Find the linearization at $x = a$.

30. $f(x) = \frac{1}{x}; a = 2$

31. $f(x) = \frac{x^2}{x-3}; a = 4$

Related Rates (Remember to organize using "know", "find", "hold", and "equation")

32. For parts a-c, assume that the radius *r* of a sphere is expanding at a rate of 30 cm/min. Determine the given rate.

a. Volume with respect to time when $r = 15$ cm.

b. Volume with respect to time at $t = 2$ min, assuming that $r = 0$ when $t = 0$.

c. Surface area with respect to time when $r = 40$ cm.

33. Two cars start moving from the same point. One travels south at 25 mi/hr, and the other travels west at 53 mi/hr. At what rate is the distance between the cars increasing 4 hours later?

For questions #34 and 35, for the function given, find intervals of increase, decrease, and *x*-coordinates of relative maximums and relative minimums (if any).

34. $f(x) = 6x - x^2$

35. $f(x) = x^3 - 12x^2 + 21x$

For questions #36 and 37, for the function given, find intervals of concavity (up and down), and *x*-coordinates of inflection points (if any).

36. $f(x) = \frac{1}{3}x^3 + x^2 + 3x$

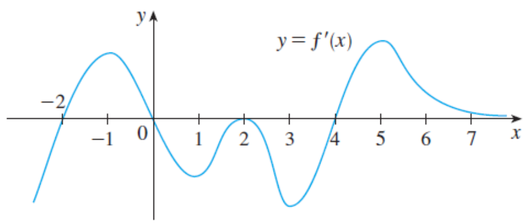
37. $f(x) = 7x^4 - 6x^2 + 1$

For questions #38 and 39, for the function given, find the absolute maximum and minimum values on the given interval. Recall the closed interval method – check endpoints!

38. $y = 2x^2 + 4x + 5; [-2, 2]$

39. $y = x + \sin x\,; [0, 2\pi]$

40.



The graph above is the graph of $f'(x)$. Answer the following questions and justify your answers.

a) When (*x*-coordinates only) does the graph of $f(x)$ have a relative maximum? Relative minimum?

b) On what interval is $f(x)$ both decreasing AND concave up?

c) When (*x*-coordinates only) does the graph of $f(x)$ have an inflection point?

OPTIMIZATION

41. Find the point on the parabola $x + y^2 = 0$ that is closest to the point (0, -3).

42. A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

43. A rectangular storage container with an open top is to have a volume of 10 m². The length of its base is twice the width. Material for the base costs \$13 per square meter, and the material for the sides costs \$10 per square meter. Find the cost of the materials for the cheapest such container. Round the result to the nearest cent.

44. Use the first and second derivative tests to supply the required information for the curve

$y = \frac{x}{x+4}$. **DISAC! (*Read the requirements for sign charts below)**

a. Intervals of increase and decrease of y
b. Values of x where y has a local maximum or minimum. Then find the max/min.
c. Intervals where y is concave up and concave down
d. Points of inflection
e. Vertical, horizontal, slant asymptotes
f. x and y intercepts (if convenient)
g. Is the function even, odd, or neither?
h. Sketch a graph based on the information in a – g.

Important Note from AP Reader: When using sign charts, they must be labeled y' and y'' **AND** the sign chart **must accompanied by a written interpretation** such as, "the curve has a maxium at x = 3 because it is increasing on the interval (0, 3) and decreasing on (3, 5)."

33. Sketch the direction field for the differential equation $y' = x^2 + y^2 - 1$. Then find a solution curve that passes through the origin.